(Annexure-I)

UNIVERSITY OF DELHI

M.A./M.Sc. MATHEMATICS
SEMESTER SYSTEM

TWO-YEAR FULL TIME PROGRAMME

RULES, REGULATIONS AND COURSE CONTENTS

Department of Mathematics
Faculty of Mathematical Sciences
University of Delhi
Delhi-110007

2012

Syllabus applicable for the students seeking admission to the M.A./M.Sc. Mathematics course in the Academic Year 2012-2013

University of Delhi

Examination Branch

Date: 01.05.2012

Check List of Course Evaluation for AC Consideration

S.No.	Parameters	Status
1.	Affiliation	√
2.	Programme Structure	√
3.	Codification of Papers	√
4.	Scheme of Examinations	√
5.	Pass Percentage & Promotion Criteria	√
6.	Reappearance in Passed Papers	√
7.	Division Criteria	√
8.	Qualifying Papers	X
9.	Span Period	√
10.	Attendance Requirements	X
11.	Course Content of Each Paper	√
12.	List of Readings	√

MASTER OF ARTS / SCIENCE
(MATHEMATICS)

TWO-YEAR FULL-TIME PROGRAMME

AFFILIATION

The proposed programme shall be governed by the Department of Mathematics, Faculty of Mathematical Sciences, University of Delhi, Delhi-110007.

PROGRAMME STRUCTURE

The master's programme in Mathematics is divided into two parts as hereunder. Each part will consist of two semesters.

Part I	First Year	Semester – 1	Semester – 2
Part II	Second Year	Semester – 3	Semester – 4

The courses prescribed for various semesters shall be the following:

PART I: Semester-1

Math 101 – Complex Analysis
Math 102 – Functional Analysis
Math 103 – Field Theory
Math 104 – Differential Equations

PART I: Semester-2

Math 201 – Topology
Math 202 – Measure and Integration
Math 203 – Module Theory
Math 204 – Fluid Dynamics

PART II: Semester-3

Math 301: Any course out of the following

(i) Advanced Complex Analysis
(ii) General Measure Theory
(iii) General Topology

Math 302: Any course out of the following

(i) Fourier Analysis
(ii) Matrix Analysis
(iii) Theory of Operators
(iv) Computational Methods for Ordinary Differential Equations

Math 303: Any course out of the following

(i) Introduction to Algebraic Topology
(ii) Advanced Group Theory
(iii) Representation of Finite Groups
(iv) Computational Fluid Dynamics

Math 304: Any course out of the following

(i) Coding Theory
(ii) Mathematical Programming
(iii) Graph Theory
(iv) Methods of Applied Mathematics

PART II: Semester-4

Math 401: Any course out of the following

(i) Differential Geometry
(ii) Commutative Algebra
(iii) Calculus on R^n

Math 402: Any course out of the following

(i) Abstract Harmonic Analysis
(ii) Advanced Functional Analysis
(iii) Theory of Frames
(iv) Operators on Hardy-Hilbert Spaces
(v) Computational Methods for Partial Differential Equations

Math 403: Any course out of the following

(i) Homology Theory
(ii) Theory of Non-commutative Rings
(iii) Algebraic Number Theory
(iv) Advanced Fluid Mechanics

Math 404: Any course out of the following

(i) Advanced Coding Theory
(ii) Optimization Techniques and Control Theory
(iii) Cryptography

NOTES : (1) Each course will have 5 credits: 4 lectures, 1 discussion and 1 tutorial per week.

(2) In the beginning of the respective semesters, the department will announce the list of elective courses which will be offered during the semester depending upon the availability of lecturers and the demand of electives.

SCHEME OF EXAMINATION

1. English shall be the medium of instruction and examination.
2. Examinations shall be conducted at the end of each semester as per the Academic Calendar notified by the University of Delhi.
3. Each course will carry 100 marks and have two components: Internal Assessment 30% marks and End-Semester Examination 70% marks.
4. The system of evaluation shall be as follows:

   4.1 Internal assessment will be based on classroom participation, seminar, term courses, tests, quizzes. The weightage given to each of these components shall be decided and announced at the beginning of the semester by the individual teacher responsible for the course. No special classes will be conducted for a student during other semesters, who fails to participate in classes, seminars, term courses, tests, quizzes and laboratory work.

   4.2 The remaining 70 marks in each paper shall be awarded on the basis of a written examination at the end of each semester. The duration of written examination for each paper shall be three hours.

5. Examinations for courses shall be conducted only in the respective odd and even Semesters as per the Scheme of Examinations. Regular as well as Ex-students shall be permitted to appear / re-appear / improve in courses of Odd Semesters only at the end of Odd Semester and courses of Even Semesters only at the end of Even Semesters.

PASS PERCENTAGE & PROMOTION CRITERIA

(a) The minimum marks required to pass any paper in a semester shall be 40% in theory and 40% in Practical, wherever applicable. The student must secure 40% in the End Semester Examination and 40% in the total of End Semester Examination & Internal Assessment of the paper for both theory & practical separately.

(b) No student will be detained in I or III Semester on the basis of his / her performance in I or III Semester examination; i.e. the student will be promoted automatically from I to II or III to IV Semester respectively.

(c) A student shall be eligible for promotion from 1st year to 2nd year of the course provided he / she has passed 50% papers of I and II Semester taken together. However, he / she will have to clear the remaining paper/s while studying in the 2nd year of the programme.

(d) Students who do not fulfill the promotion criteria (c) above shall be declared fail in the part concerned. However, they shall have the option to retain the marks in the papers in which they have secured Pass marks as per Clause (a) above.

(e) A student who has to reappear in a paper prescribed for Semester I/III may do so only in the odd Semester examinations to be held in November/December. A student who has to reappear in a paper prescribed for Semester II/IV may do so only in the even Semester examinations to be held in April/May.

REAPPEARANCE IN PASSED PAPERS

(a) A student may reappear in any theory paper prescribed for a semester, on foregoing in writing her / his previous performance in the paper(s) concerned. This can be done once only in the immediate subsequent semester examination only (for example, a student reappearing in a paper prescribed for Semester I examination, may do so along with the immediate next Semester III examinations only).

(b) A candidate who has cleared the papers of Part II (III & IV Semester) may reappear in any paper of Semester III or IV only once, at the immediate subsequent examination on foregoing in writing his / her previous performance in the paper(s) concerned, within the prescribed span period.

(Note: The candidate of this category will not be eligible to join any higher course of study)

(c) In the case of reappearance in a paper, the result will be prepared on the basis of candidate's current performance in the examination.

(d) In the case of a candidate, who opts to re-appear in any paper(s) under the aforesaid provisions, on surrendering her / his earlier performance but fails to re-appear in the paper(s) concerned, the marks previously secured by the candidate in the paper(s) in which he / she has failed to re-appear shall be taken into account while determining her/ his result of the examination held currently.

(e) Reappearance in practical examinations, dissertation, project and field work shall not be allowed.

(f) A student who reappears in a paper shall carry forward the internal assessment marks, originally awarded.

DIVISION CRITERIA

A student who passes all the papers prescribed for Parts I & II examinations would be eligible for the degree. Such a student shall be categorized on the basis of the combined result of Parts I & II examinations as follows:-

60% or more	First Division
50% or more but less than 60%	Second Division
40% or more but less than 50%	Third Division

SPAN PERIOD

No student shall be admitted as a candidate for the examination for any of the semesters after the lapse of four years from the date of admission to Semester-1 of the Master's programme in Mathematics.

Math 101 - Complex Analysis

Analytic functions as mappings, conformal mappings, Mobius transformations, branch of logarithm.

Riemann-stieltjes integrals, power series representation of analytic functions, zeros of analytic functions, maximum modulus theorem, Cauchy's theorem and integral formula on o pen subsets of C, the homotopic version of Cauchy's theorem and simply connectedness, counting zeros, open mapping theorem, Goursat'stheorem, maximum principle, schwarz's lemma.

Classification of singularities, Laurent series , residues, contour integration, argument principle, Rouche's theorem.

References:

[1] L.V. Ahlfors, Complex Analysis, McGraw Hill Co., New York, 1988.

[2] J. B. Conway, Functions of one complex variable, Narosa, Delhi, 2000.

[3] T. W. Gamelin, Complex Analysis, Springer-Verlag, 2008.

[4] S. Lang, Complex Analysis, Springer-Verlag, 2003.

Math 102 - Functional Analysis

Normed spaces, Banach spaces, properties of normed spaces, finite dimensional normed spaces and subspaces, compactness and finite dimension, linear operators, bounded and continuous linear operators, linear functionals, linear operators and functionals on finite dimensional spaces, normed spaces of operators, dual spaces.

Inner product spaces, Hilbert spaces, properties of inner product spaces, orthogonal complements and direct sums, orthonormal sets and sequences, series related to orthonormal sequences and sets, total orthonormal sets and sequences, representation of functionals on Hilbert spaces, Hilbert-adjoint operator, self-adjoint, unitary and normal operators.

Hahn-Banach theorems for real/complex vector spaces and normed spaces, application to bounded linear functionals on C[a,b], adjoint operators, reflexive spaces, category theorem, uniform boundedness theorem, strong and weak convergences, open mapping theorem, closed graph theorem.

Spectrum of an operator and its non-emptiness.

References:

[1] Erwin Kreyszig, Introductory Functional Analysis with Applications, John Wiley and Sons (Asia), Pvt. Ltd., 2006.

[2] George Bachman and Lawrence Narici, Functional Analysis, Dover, 2000.

[3] John B. Conway, A course in Functional Analysis, second edition, Springer-Verlag, 2006.

[4] Martin Schechter, Principles of Functional Analysis, second edition, AMS Book store, 2002.

[5] V.S. Sunder, Functional Analysis, Spectral Theory, Birkhauser Texts, Basel, 1997.

Math 103 - Field Theory

Fields and their extensions, splitting fields, the algebraic closure of a field, separability, automorphisms of field extensions, the fundamental theorem of Galois theory, roots of unity, finite fields, primitive elements; Galois theory of equations, the solution of equations by adicals.

References:

[1] P M. Cohn, Classic Algebra, John Wiley & Sons Ltd., 2000.
[2] P M. Cohn. Basic Algebra, Springer International Edition, 2003
[3] N Jacobson, Basic Algebra I and II, Hindustan Publishing Co., 1989.
[4] T.W Hungerford, Algebra, Springer-Verlag, 1981.

Math 104 - Differential Equations

Existence and uniqueness of solution of ordinary differential equation of first order. Picard's method. Existence theorem in complex plane. Existence and uniqueness theorem for simultaneous differential equations of first order. Existence and uniqueness theorem for ordinary differential equation of higher order. Sturm comparison and separation theorems, Homogeneous linear systems, Nonhomogeneous Linear systems, linear systems with constant coefficients.Two point boundary value problems, Greens function, construction of Green functions, Sturm-Lioville systems, Eigen values and Eigen functions.

Stability of autonomous system of differential equations, critical point of an autonomous system and their classification as stable, asymptotically stable, strictly stable and unstable.Stability of linear systems with constant coefficients. Linear plane autonomous systems, perturbed systems. Method of Lyapunov for nonlinear systems.

Solution of PDEs by method of integral transforms (Laplace and Fourier).Boundary value problems, Maximum and minimum principles, Uniqueness and continuity Theorems. Laplace equation in two-dimensions, Dirichlet and Neumann problem for half plane, Dirichlet and Neumann problem for a circle, Green's function for Laplace equation in two dimensions, Dirichlet problem for sphere and semi-infinite space, Greens function for three-dimensional Laplace equation.

Wave equation, Helmholtz's first and second theorems. Green's function for wave equation. Duhamel's principles for wave equation.

Diffusion equation, Solution of initial boundary value problems for the diffusion equation, Green's function for diffusion equation.
Duhamel's principles for heat equation.

References:

1. G.F.Simmons: Ordinary Differential Equations with applications and Historical notes *McGraw-Hill, 1991.*
2. Ian Sneddon, Elements of Partial Differential Equations, *McGraw-Hill* 1986
3. Tyn-MynT.U, Ordinary Differential Equations, Elsevier North-Holland, 1978
4. Tyn MynT.U, Linear Partial Differential Equations for Scientists and Engineers, Birkhauser, 2007.
5. S.L.Ross, Differential Equation, Wiley India, 2004

Math 201 - Topology

Topological spaces, interior, closure and boundary of a set, basis and subbasis for a topology, order topology, subspaces, continuous functions, homeomorphism, product topology, metrizability of products of metric spaces, quotient topology. Convergence: sequences and nets. T_1 and T_2 separation axioms, Connectedness, components, local connectedness, path connectedness, path components, local path connectedness. First countability, second countability, separability and Lindelöf conditions. Compactness, Bolzano-Weierstrass property, sequential compactness, local compactness and one-point compactification.

References:

[1] G. Bredon, Topology and Geometry, Springer-Verlag, 2005.

[2] J. Dugundji, Topology, Allyn and Bacon, 1970.

[3] J.L. Kelley, General Topology, Springer–Verlag, 2005.

[4] J. R. Munkres, Topology, second edition, Pearson Education, 2003.

[5] S. Willard, General Topology, Dover Publications, inc. N.Y., 2004..

Math 202 - Measure and Integration

Lebesgue outer measure, measurable sets, regularity, measurable functions, Borel and Lebesgue measurability, non-measurable sets

Integration of nonnegative functions, the general integral, integration of series, Riemann and Lebesgue integrals.

Functions of bounded variation, Lebesgue differentiation theorem, differentiation and integration, absolute continuity of functions.

Measures and outer measures, measure spaces, integration with respect to a measure.

The L^p spaces Hölder and Minkowski inequalities, completeness of L^p spaces, Convergence in measure, almost uniform convergence, Egorov's theorem.

References :

[1] G. De Barra, Measure Theory and Integration, Wiley Eastern Ltd. 1981.

[2] E. Hewitt and K. Stromberg, Real and Abstract Analysis, Springer, Berlin, 1988.

[3] H. L. Royden, Real Analysis, Pearson, 2008.

Math 203- Module Theory

Modules, Basic Concepts, Direct product and Direct sums, Exact sequences, Split exact sequences, Five lemma, Free modules, Modules over P.I.D., Nakayama lemma, Tensor product of modules, Categories and Functors, Hom functors, Semi simple modules, Projective and Injective modules, Baer's criterion, Divisible modules.

References:

1. P.M. Cohn, Classic Algebra, John Wiley & Sons Ltd., 2000.
2. P.M. Cohn, Basic Algebra, Springer International Edition, 2003.
3. D. S. Dummit & R.M. Foote, Abstract Algebra, Wiley India Pvt. Ltd.
4. T.W. Hungerford, Algebra, Springer-Verlag, 1981.
5. N. Jacobson, Basic Algebra, Volume II, Hindustan Publishing Co., 1989.

Math 204 - Fluid Dynamics

Classification of fluids, the continuum model, Eulerian and Lagrangian approach of description. Differentiation following fluid motion. Irrotational flow,vorticity vector, equi-potential surfaces. Streamlines, pathlines, streak lines of the particles, stream tube and stream surface. Mass flux density, conservation of mass leading to equation of continuity. (Euler's form.), Origin of forces in fluid. Conservation of momentum and its mathematical formulation: Euler's form. Integration of Euler's equation under different conditions. Bernoulli's equation, steady motion under conservative body forces. Boundary surface.

Theory of irrotational motion, Kelvin's minimum energy and circulation theorem, potential theorems. Some two and three dimensional flows, sources, sinks, doublets and vortices ,their images with respect to a plane and sphere. Milne –Thompson circle theorem, Butlers sphere theorem, Kelvin's inversion theorem and Weiss's sphere theorem. Axi-symmetric flows and stream function. Motion of cylinders and spheres. Two dimensional flows of irrotational, incompressible fluids, complex potential and its applications to two dimensional singularities. Blasius theorem, D'Alembert's paradox

Viscous flow, stress and strain analysis. Stokes hypothesis, The Navier-Stokes equations of motion. Some exactly solvable problems in viscous flows, steady flow between parallel plates, Poiseuille flow, steady flow between concentric rotating cylinders.

REFERENCES

1. P.K. Kundu and I.M. Cohen, Fluid Mechanics, Academic Press,2005.
2. L.M.MilneThomson, Theoretical Hydrodynamics, The Macmillan company, USA,1969.
3. N.E.Neill and F. Chorlton, Ideal and incompressible fluid dynamics, Ellis Horwood Ltd, 1986.
4. N.E.Neill and F. Chorlton, Viscous and compressible fluid dynamics, Ellis Horwood Ltd, 1986.
5. D.E.Rutherford: Fluid Dynamics, Oliver and Boyd Ltd, London.1978.
6. F.Chorlton: text book of fluid dynamics, CBS,2004

Math 301 (i) - Advanced Complex Analysis

Hadamard's three circles theorem, Phragmen-Lindelöf theorem.

The space of continuous functions C(G, Ω), spaces of analytic functions, Hurwitz's thorem, Montel's theorem, spaces of meromorphic functions, Riemann mapping theorem, Weierstrass' factorization theorem, factorization of the sine function, the gamma function, the Riemann zeta function, the Riemann functional equation.

Runge's theorem, simply connected regions, Mittag-Leffler's theorem.

Harmonic functions, maximum and minimum principles, harmonic functions on a disk, Harnack's theorem, sub-harmonic and super-harmonic functions, maximum and minimum principles, Dirichlet problem, Green's function.

References :

[1] J. B. Conway, Functions of one complex variable, Narosa Publishing House, New Delhi, 2000.

[2] T. W. Gamelin, Complex Analysis, Springer-Verlag, 2008.

[3] L. Hahn and B. Epstein, Classical Complex Analysis, Jones and Bartlett, India, New Delhi, 2011.

[4] S. Lang, Complex Analysis, Fourth edition, Addison Wesley, 1999.

Math 301 (ii) – General Measure Theory

Signed measures, complex measures, Hahn decomposition theorem, Jordan decomposition theorem, mutually singular measures. Radon-Nikodym theorem, Lebesgue decomposition, Riesz representation theorem, extension theorem (Caratheodory), Lebesgue-Stieltjes integral, cummulative distribution function, product measures, Fubini's theorem. Tonelli theorem, Differentiation and integration.

Baire sets, Baire measures, continuous functions with compact support, regularity of measures on locally compact spaces, integration of continuous functions with compact support, Riesz · Markov representation theorem.

References:

[1] J.M. G Fell and R. S. Doran, Representation of * - algebras, locally compact groups and Banach *-Algebraic Bundles, Vol I, Academic press Inc , 1988.

[2] P. R. Halmos, Measure Theory, East-West Press Private Ltd., 1978.

[3] E. Hewitt and K.A Ross, Abstract Harmonic Analysis. Vol.I, Springer Verlag, fourth edition, 1993.

[4] H.L. Royden, Real Analysis, Pearson, 2008.

Math 301 (iii) - General Topology

Regularity, complete regularity, the Stone–Čech compactification, normality, Urysohn lemma, Urysohn metrization theorem and Tietze extension theorem, paracompactness, characterizations of paracompactness in regular spaces, Function spaces, topology of point-wise convergence, compact-open topology, exponential law, topologies of uniform convergence, compact convergence and continuous convergence, equicontinuity, Arzela-Ascoli theorem.

References:

[1] J. Dugundji, Topology, Allyn and Bacon, 1970.
[2] R. Engelking, General Topology, Heldermann, 1989.
[3] J.L. Kelley, General Topology, Springer-verlag, 2005.
[4] J.R. Munkres, Topology, Second Edition, Pearson Education, 2003.
[5] S. Willard, General Topology, Dover Publications, Inc. N.Y., 2004.

Math 302 (i) – Fourier Analysis

Convergence and divergence of Fourier series. Fejer's theorem Approximate identities. The classical kernels : [Fejer's, Poisson's and Dirichlet's Summability in norm and pointwise summability], Fatou's Theorem. The inequalities of Hausdorff and Young. Examples of conjugate function series. The Fourier transform. Kernels on R. Basic properties of topological groups, separation properties, subgroups, quotient groups and connected groups. Notion of Haar Measure on topological groups with emphasis on R, T, Z and some simple matrix groups. L^1 (G) and convolution with special emphasis on L^1 (R), L^1 (T), L^1 (Z). Plancherel theorem on abelian groups. Plancherel measure on R, T, Z. Maximal ideal space of L^1(G) (G an abelian topological group).

References:

[1] H. Helson, Harmonic Analysis, Addison-Wesley, 983, Hindustan Pub. Co., 1994.

[2] E. Hewitt and K.A. Ross, Abstract Harmonic Analysis Vol. I, Springer-Verlag. 1993.

[3] Y. Katznelson, Introduction to Harmonic Analysis, John Wiley, 2004.

Math 302(ii) - Matrix Analysis

Closed subgroups of General Linear group. Examples and their compactness and connectedness. Norms for vectors and matrices. Geometric properties of vector norms. Matrix norms. Error in inverses and solution of linear systems. Matrix exponential. Location and perturbation of eigen values. Gersgorin discs. Other inclusion regions.Positive definite matrices. Polar form and singular value decomposition. Applications of singular value decomposition. The schur product theorem. Inequalities for positive definite matrices.Positive matrices and Perron's theorem. Majorisation and Doubly Stochastic Matrices.

References:

1. R. Bhatia, Matrix Analysis, Springer Verlag, 1996
2. B.C.Hall, Lie groups, Lie Algebras, and Representations An Elementary Introduction, Springer Verlag, 2003
3. R.A.Horn and C.R.Johnson, Matrix Analysis, Cambridge University Press, 1994.

Math 302 (iii) – Theory of Operators

Spectrum, Basic concepts, point, continuous and residue spectrum, approximate point spectrum and compression spectrum, spectral mapping theorem for polynomials, Uniform, strong and weak operator convergences on the space of bounded linear operators.

Compact linear operators, properties of compact operators, adjoint of compact operators, Spectral properties of compact operators, Fredholm theory of compact operators and operator equations.

Spectral properties of self-adjoint linear operators, Positive operators and their properties, projection operators and their properties, spectral representation of a self-adjoint compact operator, application to the integral operators, spectral family of self-adjoint operators, spectral representation of a self-adjoint operator, continuous functions of self-adjoint operators, properties of the spectral family of a bounded self-adjoint operator.

Polar decomposition, singular values, trace class operators, trace norm and trace, Hilbert-Schmidt operators.

References

(1) Rajendra Bhatia, Notes on Functional Analysis, Texts and Reading in Mathematics, Hindustan Book Agency (2009).
(2) J. B. Conway, A Course in Functional Analysis, Springer (1990).
(3) Y. Eidelman, V. Milman, A. Tsolomitis, Functional Analysis An Introduction, American Mathematical Society (2004).
(4) E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley and Sons (2001).
(5) P. D. Lax, Functional Analysis, John Wiley and Sons (2002).

Math 302 (iv) -
Computational Methods for Ordinary Differential Equations

Initial Value Problems (IVPs) for the system of ordinary differential equations (ODEs); Difference equations; Numerical Methods; Local truncation errors; Stability analysis; Interval of absolute stability; Convergence and consistency.

Single-step Methods: Taylor series method; Explicit and Implicit Runge-Kutta methods and their stability and convergence analysis; Extrapolation method; Runge-Kutta method for the second order ODEs; Stiff-system of differential equations.

Multi-step Methods: Explicit and Implicit multi-step methods; General linear multi-step methods and their stability and convergence analysis; Adams-Moulton method; Adams-Bashforth method; Nystorm method; Multi-step methods for the second order IVPs.

Boundary Value Problems(BVPs): Two point non-linear BVPs for second order ordinary differential equations; Finite difference methods; Convergence analysis; Difference scheme based on quadrature formula; Difference schemes for linear eigen value problems; Mixed boundary conditions; Finite element methods; Assemble of element equations; Variational formulation of BVPs and their solutions; Galerikin method; Ritz method; Finite element solution of BVPs;

Note: 1. This course consists of two parts:

(i) Final theory examination carries 70 marks

(ii) Internal assessment examination carries 30 marks

Note: 2. Use of scientific calculator is allowed in theory examination

Books Recommended:

[1] J.C. Butcher, Numerical Methods for Ordinary Differential Equations, John Wiley & Sons, New York, 2003.

[2] J.D. Lambert, Numerical Methods for Ordinary Differential Systems: The Initial Value Problem, John Wiley and Sons, New York, 1991.

[3] K. Atkinson, W.Han and D.E. Stewart, Numerical Solution of Ordinary Differential Equations, John Wiley, New York, 2009.

Math 303 (i) - Introduction to Algebraic Topology

Homotopic maps, homotopy type, retraction and deformation retract. Fundamental group. Calculation of fundamental groups of n-sphere, n ≥ 1, the cylinder, the torus, and the punctured plane. Applications: the Brouwer fixed-point theorem, the fundamental theorem of algebra, free products, free groups, Seifert–Van Kampen theorem and its applications.

Covering projections, the lifting theorems, relations with the fundamental group, universal covering space. The Borsuk-Ulam theorem, classification of covering spaces.

References:

[1] G.E. Bredon, Geometry and Topology, Springer-Verlag, 2005.

[2] W.S. Massey, A Basic Course in Algebraic Topology, Springer-Verlag, 1991.

[3] J.J. Rotman, An Introduction to Algebraic Topology, Springer-Verlag, 2004.

[4] E.H. Spanier, Algebraic Topology, Springer-Verlag, 1989.

Math 303 (ii) – Advanced Group Theory

Regular, coset and conjugate representations, G-sets and applications, normal series, refinements, composition series, Zassenhaus Lemma, Schreier's theorem on refinements, Jordan-Holder theorem.

Solvable groups, derived series, supersolvable groups, minimal normal subgroups, Hall's theorems, Hall subgroup, p-complements, central series, nilpotent groups, Schur's theorem, Fitting subgroup, Jacobi identity, Three subgroup Lemma, Frattini subgroup, Burnside basis theorem.

Indecomposable groups, group with ascending and descending chain conditions, Fitting's Lemma, Krull-Schmidt theorem, subnormal subgroups, semidirect products, Schur-Zassenhaus lemma, transfer and Burnside normal complement theorem and its consequences.

References:

1. T.W. Hungerford, Algebra, Springer-Verlag, New York, 1981.
2. D.J.S. Robinson, A course in the theory of groups, Springer-Verlag, New York, 1996.
3. J.S. Rose, A course on group theory, Dover Publications, New York, 1994.
4. J.J. Rotman, An Introduction to the Theory of Groups, Springer-Verlag, New York, 1995.
5. M. Suzuki, Group Theory-I, Springer-Verlag, Berlin, 1982.

Math 303 (iv) - Computational Fluid Dynamics

Mathematical description of the physical phenomena. Governing equations- mass, momentum, energy, species. General form of the scalar transport equation, Elliptic, parabolic and hyperbolic equations. Methods for deriving discretization equations by finite difference and finite volume method. Method for solving discretization equations.One-dimensional and two dimensional Diffusion Equation, unsteady diffusion, explicit, implicit and Crank-Nicolson scheme. Two dimensional conduction, accuracy, stability and convergence.

Convection and Diffusion- Steady one-dimensional convection and diffusion, upwind, exponential, hybrid, power, QUICK scheme, Two-dimensional convection-diffusion, accuracy of upwind scheme; false diffusion and dispersion, Boundary conditions. Flow field calculation, pressure-velocity coupling, vorticity-stream function formulation, staggered grid, SIMPLE family of algorithms.

Numerical methods for radiation- Radiation exchange in enclosures composed of diffuse gray surfaces, Finite volume method for radiation.

References

1. John D. Anderson, Computational Fluid Dynamics, *McGraw-Hill*, 1995.
2. D.A. Anderson J.C. Tannehill and Richard H. Pletcher, Computational Fluid Mechanics and Heat Transfer, Taylor and Francis, 1997.
3. Ferziger J H, Peric M, Computational Methods for Fluid dynamics, Spriger, 2001.
4. Suhas V. Patankar, Numerical Heat Transfer and Fluid Flow: Taylor and Francis, 2004.
5. Versteeg, H.K. and Malalasekera,W An introduction to Computational Fluid Dynamics: The Finite volume method, Pearson, 2007.
6. P.S. Ghoshdastidar, Computer Simulation of flow and Heat transfer, Tata-McGrahill Ltd, New Delhi(1998).

Math 304 (i) – Coding Theory

The communication channel, the coding problem, types of codes, block codes, error-detecting and error-correcting codes, linear codes, the Hamming metric, description of linear block codes by matrices, dual codes, standard array, syndrome, step-by-step decoding, modular representation, error-correction capabilities of linear codes, bounds on minimum distance for block codes, Plotkin bound, Hamming sphere packing bound, Varshamov-Gilbert-Sacks bound, bounds for burst-error detecting and correcting codes, important linear block codes, Hamming codes, Golay codes, perfect codes, quasi-perfect codes, Reed-Muller codes, codes derived from Hadamard matrices, product codes, concatenated codes.

References:

[1] Raymond Hill, A First Course in Coding Theory, Oxford University Press, 1990.

[2] W.W. Peterson and E.J. Weldon Jr., Error-Correcting Codes, M.I.T. Press, Cambridge, Massachusetts, 1972.

[3] Man Young Rheo, Error Correcting Coding Theory, McGraw Hill Inc., 1989.

[4] F.J. MacWilliams and N.J.A. Sloane, The Theory of Error Correcting Codes, North Holland publishing company, 2006.

Math 304 (ii) -Mathematical Programming

Existence theorems, First order optimality conditions and second order optimality conditions for unconstrained optimization problems, Convex functions, Optimization on convex sets, Separation theorem, Quasiconvex functions, Pseudoconvex functions.

Fritz John and Kuhn Tucker optimality conditions for constrained nonlinear programming problems, Second order optimality conditions for constrained problem, Lagrangian saddle points, Lagrangian duality in convex programming.

Quadratic programming, Wolfe's method as application of Kuhn Tucker conditions, Convex simplex method, Penalty function method.

References

1. O. Güler, **Foundation of Optimization**, Springer, 2010.
2. R.K. Sundaram, **A First Course in Optimization Theory**, Cambridge University Press, 2009.
3. M.S. Bazaraa, H.D. Sherali and C.M. Shetty, **Nonlinear Programming: Theory and Algorithms**, John Wiley & Sons, 2006.

Math 304 (iii) – Graph Theory

Graphs: Vertices of graphs, walks and connectedness, degrees, operations on graphs, blocks, cut-points, bridges and blocks, block graphs and cut-point graphs.

Trees: Elementary properties of trees, centers and centroids, block-cut point trees, independent cycles and co-cycles.

Connectivity and Traversability: Connectivity and line connectivity, Menger's theorems, Eulerian graph, Hamiltonian graphs.

Planarity and Coloring: Planar graphs, outer planar graphs, Kuratowski's theorem, dual graphs, chromatic number, five color theorem.

References:

[1] R. Balakrishnan and K. Ranganathan, A Text Book of Graph Theory, Springer, 2000.

[2] B. Bollobas, Modern Graph Theory, Springer, 2002.

[3] G. Chartrand and L. Lesniak, Graphs and Digraphs, 4th Edit., Chapman & Hall (CRC), 2005.

[4] F. Harary, Graph Theory, Narosa Publishing House, New Delhi, 2001.

[5] R.J. Wilson, Introduction to Graph Theory, 4th Edit., Addison Wesley, 1996.

Math 304 (iv) - Methods of Applied Mathematics

Perturbation methods, regular perturbations, singular perturbations, applications of perturbation methods to fluid mechanics. WKB approximations.

Volterra integral equations, relationship between linear differential equations and Volterra integral equations, resolvent kernel of Volterra integral equation, solution of integral equations by resolvent kernel. The method of successive approximations, convolution type equations. Solutions of integral equations with the aid of Laplace transformation.

Fredholm integral equations, iterated kernels, constructing the resolvent kernel with the aid of iterated kernels. Integral equations with degenerate kernels, characteristic numbers and eigen functions, solution of homogeneous integral equations with degenerate kernel, non homogeneous symmetric equations, Fredholm alternative.

Extrema of functional, the variation of a functional and its properties. Euler's equation and its generalization, sufficient conditions for the extremum of a functional, conditional extremum, moving boundary problems, Ritz method.

Stability and bifurcation: basic ideas and one dimensional problem.

REFERENCES:

1. M. Gelfand and S.V. Fomin, Calculus of variations, Prentice Hall, Inc., 2000.
2. F.B. Hildebrand, Methods of applied mathematics, Dover Publication, 1992.
3. M.L. Krasnov, Problems and exercises integral equations, Mir Publication Moscow, 1971.
4. D. Logan: Applied mathematics: A contemporary approach, John Wiley and Sons, New York, 1997.

Math 401 (i) - Differential Geometry

Graph and level sets, vector fields, the tangent space, surfaces, orientation, the Gauss map, geodesics, parallel transport, the Weingarten map, curvature of plane curves, arc length and line integrals, curvature of surfaces, parametrized surfaces, surface area and volume, surfaces with boundary, the Gauss-Bonnet Theorem.

References

[1] Wolfgang Kuhnel: Differential Geometry – curves-surfaces-Manifolds. Second Edition, 2006, AMS.

[2] A: Mishchenko and A. Formentko. A course of Differential Geometry
and topology) Mir Publishers Moscow, 1988.

[3] Andrew Pressley: Elementary Differential Geometry. SUMS (Springer), 2001 (Ist Indian Repring 2004).

[4] I.A. Thorpe: Elementary Topics in Differential Geometry. Springer, 1979 (Ist Indian Reprint 2004).

Math 401(ii) -Commutative Algebra

Extension and Contraction of ideals, Prime spectrum of Rings, Jacobson radical of a ring, Prime avoidance lemma, Rings of formal power series, Restriction and extension of scalars.

Localisation, Local properties, Extended & contracted ideals in rings of fractions, Primary decomposition, First and second uniqueness theorem of primary decomposition, Chain conditions, Noetherian rings, Hilbert's Basis Theorem, Primary decomposition in Noetherian rings, Artin rings, Structure theorem for Artin rings.

Integral dependence, Going up theorem, Going down theorem, Integrally closed domains, Valuation rings, Hilbert's Nullstellensatz theorem, Discrete valuation rings, Dedekind domains, Fractional ideals.

References:

[1] M.F. Athiya & I.G. Macdonald, Introduction to Commutative Algebra, Addison Wesley, 1969.

[2] Balwant Singh, Basic Commutative Algebra, World Scientific Publishing Co., 2011.

[3] D. Eisenbud, Commutative Algebra with a view towards algebraic geometry, Springer Verlag, 1995.

[4] O. Zariski & P. Samuel, Commutative Algebra, Vol. 1 & 2, Springer-Verlag, 1975.

[5] R.Y. Sharp, Steps in Commutative Algebra, Cambridge University Press, 1990

Math-401(iii) - Calculus on R^n

Thedifferentiability of functions from R^ntoR^n, directional derivatives and differentiability, chain rule, inverse function theorem and implicit function theorem.

Integration over a k-cell, primitive mappings, partition of unity, change of variables, Introduction to differential forms on R^n, basic properties of differential forms, differentiation of differential forms, change of variables in differential forms, simplexes and chains, integration of differential forms, Stokes' theorem.

References:

[1]. J.R. Munkres, Analysis on manifolds, Addison Wesley, 1991.

[2]W. Rudin, Principles of Mathematical Analysis, 3rd edition, McGraw Hill, 1986

[2].M. Spivak, Calculus on Manifolds: A Modern Approach to Classical Theorems Of Advanced Calculus, Westview Press, 1998.

Math 402 (i) - Abstract Harmonic Analysis

Introduction to representation theory of involutive Banach algebra. Unitary representation of locally compact groups, Gelfand-Rajkov theorem.

Representation of some special groups SU(2), Lorentz group, the group of linear transformations of the real line. Unitary representation of compact groups. Schur's lemma, the orthogonality relations.

Characters of finite dimensional representation. Weyl-Peter theorem, convolution of bounded regular complex measures.

The convolutive Banach algebra M(G). Fourier-Stieltjes transform. Positive definite functions. Bochner's theorem.

References:

[1] J.M.G. Fell and R.S. Doran, Representation of * Algebras. Locally compact Groups I and Banach* Algebraic Bundles. Vol. I, II, Academic Press Inc., 1988.

[2] E. Hewitt and K.A. Ross, Abstract Harmonic Analysis, Vol. I., II Springer Verlag, 1993.

[3] W. Rudin, Fourier Analysis on Groups, Interscience Publisher, 1990.

Math 402 (ii) - Advanced Functional Analysis

Introduction to topological vector spaces and locally convex spaces, linear operators, uniform boundedness principle, closed graph theorem, open mapping theorem, Hahn-Banach theorem, extreme points and Krein-Milman theorem.

Geometry of Banach spaces: vector measures, Radon-Nikodym property and geometric equivalents, Choquet theory. Weak compactness and Eberlein-Smulian theorem, Schauder basis.

References:

[1] J. Diestel and J. J. Uhl, Jr., Vector measures. Mathematical Surveys, No. 15. American Mathematical Society, 1998.

[2] N. Dunford and J. T. Schwartz, Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space. Interscience Publishers John Wiley & Sons, 1963.

[3] Walter Rudin, Functional analysis. International Series in Pure and Applied Mathematics. McGraw-Hill, Inc., 1991.

[4] K. Yosida, Functional analysis. Grundlehren der Mathematischen Wissenschaften, 123. Springer-Verlag, 2008.

Math. 402(iii)-Theory of Frames

Haar Wavelet Analysis: Haar Wavelets, Basic properties of the Haar scaling function, Haar decomposition and reconstruction algorithms, Filter and Diagrams. Comparison of Haar series with Fourier series.

Multiresolution Analysis: The Scaling Relation, The associated wavelet and wavelet spaces, Decomposition and reconstruction formulas, The decomposition and reconstruction algorithm, Processing a signal. Fourier Transform Criteria.

Frames: Bessel sequences in Hilbert spaces, Riesz bases, The Gram matrix, Gabor bases,Frames and their properties, Frame sequence, Frames and Riesz bases, Bases in Banach spaces, Limitations of bases, Frames and operators, Frames and bases, Characterization of frames, The dual frames, Tight frames, Continuous frames, frames and signal processing. Conditions for a frame being a Reisz basis, Frames contaiing a Riesz basis, Frames which does not contain a basis, Comparison between frames and Riesz bases. Frames in finite dimensional Hilbert spaces.

Wavelet versus frames: Wavelets frames and wavelet sets, Estimates of wavelet frame bounds, Admissibility Condition, Signal and Systems, Perturbation of wavelet frame.

References

[1] Ole Christensen, *An introduction to frames and Riesz bases*, *Birkhäuser* (2003)

[2] Ole Christensen, *Frames and bases* , *Birkhäuser*(2008)

[3] Christopher Heil, *A Basis Theory Primer* (Expanded Edition), *Birkhäuser* (2011)

[4] K. Grochenig, *Foundations of time- frequency analysis*, *Birkhäuser* (2000).

[5] Albert Boggess and Francis J. Narcowich, *A first course in wavelets and Fourier analysis*, *Wiley* (2009)

[6] David F. Walnut, *An Introduction to Wavelet Analysis*, *Birkhäuser* (2002).

Math 402 (iv) - Operators on the Hardy-Hilbert Space

The Hardy-Hilbert Space: Basic definitions and properties.

The unilateral shift and factorization of functions: Shift operators. Invariant and reducing subspaces. Inner and outer factorization. Balschke factors. Singylar inner functions. Outer functions.

Toeplitz operators: Basic properties of Toeplitz operators. Spectral structure.

Hankel operators: Bounded Hankel operators. Hankel operators of finite rank. Compact Hankel operators. Self adjointedness and normality of Hankel operators. Relation between Hankel and Toeplitz operators.

References:

[1] R. G. Douglas, Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics 179, Springer, 1998.

[2] R.A. Martinez-Avedano & P. Rosenthal, An Introduction to the Hardy-Hilbert Space, Graduate Texts in Mathematics 237, Springer, 2007.

[3] N. K. Nikolskii, Operators, Functions and Systems: An Easy Reading, Volume I, Mathematical Surveys and Monographs 92, American Mathematical Society, 2002.

Math 402 (v)-
Computational Methods for Partial Differential Equations

Finite difference methods for 2D and 3D elliptic boundary value problems (BVPs) of second and fourth order approximations; Finite difference approximations to Poissons equation in cylindrical and spherical polar coordinates; Solution of large system of algebraic equations corresponding to discrete problems and iterative methods (Jacobi, Gauss-Seidel and SOR); Alternating direction methods.

Different 2- and 3-level explicit and implicit finite difference approximations to heat conduction equation; Stability analysis (Energy method, Matrix method and Von-Neumann method); Compatibility, consistency and convergence of the difference methods; Difference scheme based on derivative boundary conditions and its stability condition; ADI methods for 2- & 3-D parabolic equations; Finite difference approximations to heat equation in polar coordinates.

Methods of characteristics for evolution problem of hyperbolic type; Von-Neumann method for stability analysis; Operator splitting methods for 2D and 3D wave equations; Explicit and implicit difference schemes for first order hyperbolic equations and their stability analysis; System of equations for first order hyperbolic equations; Conservative form.

Finite element methods for second order elliptic BVPs; Finite element equations; Variational problems; Triangular and rectangular finite elements; Standard examples of finite elements; Mixed finite element methods.

Note: 1. This course consists of two parts:

(i) Final theory examination carries 70 marks
(ii) Internal assessment examination carries 30 marks

Note: 2. Use of scientific calculator is allowed in theory examination

Books Recommended:

[1] J.C. Strickwerda, Finite Difference Schemes & Partial Differential Equations, SIAM publications, 2004.

[2] J.W.Thomas, Numerical Partial Differential Equations: Finite Difference Methods, Springer and Verlag, Berlin, 1998.

[3] J.W.Thomas, Numerical Partial Differential Equations: Conservation Laws and Elliptic Equations, Springer and Verlag, Berlin, 1999.

Math 403 (i) - Homology Theory

Geometric simplexes, geometric complexes and polyhedra. Simplicial maps, barycentric subdivision, simplicial approximation of continuous maps, contiguous maps, abstract simplicial complex.

Orientation of geometric complexes, homology groups. Computation of homology groups, the homology of n-sphere, $n \geq 1$. The structure of homology groups, the chain complexes, chain mappings, chain derivation, chain homotopy. The homomorphism induced by continuous maps between two polyhedra.

Singular complex and homology groups, functorial properties, the Eilenberg-Steenrod axioms of homology theory. The reduced homology groups, the Mayer-Vietoris sequence. The degree of self mappings of S^n, the Brouwer's fixed point theorem, the Euler-Poincaré theorem and Lefschetz fixed point theorem.

References:

[1] H. Agoston, Algebraic Topology, Marcel Dekker, 1976.
[2] M. A. Armstrong, Basic Topology, Springer-Verlag, 1983.
[3] F.H. Croom, Basic Concepts of Algebraic Topology, 1976.
[4] A. Dold, Lectures on Algebraic Topology, Springer-Verlaq, second edition, 1980.
[5] J.J. Rotman, An Introduction to Algebraic Topology, Springer-Verlag, 1988.

Math 403 (ii) - Theory of Noncommutative Rings

Basic terminology and examples, semisimplicity, structure of semisimple rings, Wedderburn–Artin's theorem, Jacobson radical, prime radical, prime and semiprime rings, structure of primitive rings, density theorem, direct products, subdirect sums, commutativity theorems and local rings

References:

[1] I.N. Herstein, A First Course in Noncommutative Rings, Carus Monographs of AMS 1968.

[2] Louis H. Rowen, Ring Theory, Academic Press, 1991.

[3] T.W. Hungerford, Algebra, Springer Verlag, New York, 1981.

[4] T.Y. Lam, A first course on Non-Commutative Rings, Springer-Verlag, 1991.

Math 403 (iii) – Algebraic Number Theory

Algebraic numbers, number fields, conjugates and discriminants, algebraic integers, integral bases, norms and traces, rings of integers, quadratic fields and cyclotomic fields.

Trivial factorizations, factorization into irreducibles, examples of non-unique factorization into irreducibles, prime factorization, Euclidean domains and Euclidean quadratic fields, consequences of unique factorization, the Ramanujan-Nagell theorem. Prime factorization of ideals, norm of an ideal, non-unique factorization in cyclotomic fields

Lattices of dimension *m*, the quotient torus, Minkowski's theorem, two-squares theorem, four-squares theorem, the space L^{st}. The class-group, finiteness of the class-group, unique factorization of elements in an extension ring, factorization of a rational prime, Minkowski's constants, class-number calculations.

References:

[1] K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, Springer-Verlag, 1990.

[2] S. Lang, Algebraic Number Theory, Springer-Verlag, New York Inc., 1994.

[3] D.A. Marcus, Number Fields, Springer-Verlag, New York Inc., 1987.

[4] I. N. Stewart and D. O. Tall, Algebraic Number Theory, Chapman and Hall, London, 1987.

Math 403 (iv)- Advanced Fluid Mechanics

Thermodynamics: Equation of state of a substance, First law of Thermodynamics, internal energy and specific heat of gas, entropy, Second law of thermodynamics.

Physical similarity and Dimensional Analysis: Types of physical similarity, Nondimensionalizing the basic equation of incompressible viscous fluid flow, non-dimensional parameters, Dimensional analysis and Buckingham Pi Theorem.

Gas Dynamics: Compressibility effects, Elements of wave motion in a gas, Speed of sound, Basic equation of one-dimensional compressible flow, Subsonic, sonic and supersonic flows, Isentropic gas Flow, Flow through a nozzle, Normal shock wave, oblique shock wave and their elementary analysis.

Magnetohydrodynamics: Concept, Maxwell' electromagnetic field equations, Equation of motion of a conducting fluid, MHD approximations, Rate of flow of charge, Magnetic Reynolds number and Magnetic field equation, Alfven's theorem, Magnetic body force, Ferraro's Law of isorotation.

Boundary Layer theory: Concept, Boundary layer thickness, Prandtl's boundary layer, Boundary layer on flat plate: Blassius solution, Karman's integral equation.

References

1. Text Book of Fluid Dynamics, F. Chorlton, GK Publisher,2009
2. Fluid Mechanics, P.K.Kundu, I.M.Cohen, Academic Press,2010
3. Introduction to Fluid Mechanics, R.W.Fox, P.J.Pritchard, A.T.Mcdonald. John Wiley and Sons, 2010
4. Introduction to Fluid Mecanics, G.K.Batchelor, Foundation book, New Delhi. 1994
5. Compressible Fluid Flow, S.I.Pai, New York, Ronald Press, 1954
6. Alan Jeffery, Magnetohydrodynamics, Oliver and Boyd Ltd., Edinburgh, 1966

Math 404 (i) – Advanced Coding Theory

Tree codes, convolutional codes, description of linear tree and convolutional codes by matrices, standard array, bounds on minimum distance for convolutional codes, V-G-S bound, bounds for burst-error detecting and correcting convolutional codes. The Lee metric, packing bound for Hamming code w.r.t. Lee metric. The algebra of polynomials, residue classes, Galois fields, multiplicative group of a Galois field, cyclic codes, cyclic codes as ideals, matrix description of cyclic codes, Hamming and Golay codes as cyclic codes, error detection with cyclic codes, error-correction procedure for short cyclic codes, shortended cyclic codes, pseudo cyclic codes, code symmetry, invariance of codes under transitive group of permutations, Bose-Chaudhary-Hocquenghem (BCH) codes, BCH bounds, Reed-Solomon (RS) codes, majority-logic decodable codes, majority-logic decoding, singleton bound. The Griesmer bound, maximum-distance separable (MDS) codes, generator and parity-check matrices of MDS codes, weight distribution of MDS code, necessary and sufficient conditions for a linear code to be an MDS code, MDS codes from RS codes, Abramson codes, closed-loop burst-error correcting codes (Fire codes), error locating codes.

References:

[1] E.R. Berlekamp, Algebraic Coding Theory, McGraw Hill Inc., 1984..

[2] W.C. Huffman and V. Pless, The Theory of Error Correcting Codes, Cambridge University Press, 1998.

[3] F.J. MacWilliams and N.J.A. Sloane, The Theory of Error Correcting Codes, North Holland publishing company, 2006.

[4] W.W. Peterson and E.J. Weldon Jr., Error-Correcting Codes, M.I.T. Press, Cambridge, Massachusetts, 1972.

Math 404 (ii)-Optimization Techniques and Control Theory

Functions taking values in extended reals, Proper convex functions, Subgradients, Directional derivative, Conjugate functions, Conjugate duality.

Gradient descent method, Gradient projection method, Newton's method, Conjugate gradient method.

Dynamic programming, Bellman's principle of optimality, Allocation problem, Cargo load problem, Stage coach problem.

Optimal control problem, Classical approach to solve variational problem, Pontryagin's maximum principle, Dynamic programming and maximum principle.

References

1. M. Avriel, **Nonlinear Programming: Analysis and Methods**, Dover Publications, New York, 2003.
2. O. Güler, **Foundation of Optimization**, Springer, 2010.
3. F. Hillier, G.J. Lieberman, **Introduction to Operations Research**, McGraw-Hill College, 2009.
4. D. Liberzon, **Calculus of Variations and Optimal Control Theory: A Concise Introduction**, Princeton University Press, 2012.

Math 404 (iii) – Cryptography

Secure communications, shift ciphers, affine ciphers, vigenere cipher key, symmetric key, public key, block ciphers, one time pads, secure random bit generator, linear feedback shift register sequences.

Differential cryptanalysis, modes of DES, attack on DES, advanced encryption standard.

RSA, attack on RSA, Diffie-Hellman key exchange, ElGamal public key cryptosystem, cryptographic hash function, RSA signatures, ElGamal signature, hashing and signing, digital signature algorithm.

References:

[1] Johannes A Buchmann, Introduction to Cryptography, Springer, 2000.

[2] Douglas Robert Stinson, Cryptography – Theory and Practice, Chapman Hall (CRC), 2006.

[3] Wade Trappe and Lawrence C Washington, Introduction to Cryptography with Coding Theory, Pearson Prentice Hall, 2006.

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF DELHI
DELHI 110 007

PH.D. PROGRAMME 2013-2014

1. ELIGIBILITY

i) A candidate must have obtained a Master's/M.Phil degree of the University of Delhi, or any other recognized University, or any degree recognized as equivalent to Master's/M.Phil degree Mathematics. She/he must have obtained either a minimum of 50% marks or equivalent grading in the M.Phil degree or a minimum of 55% marks or equivalent grading in the Master's degree.

ii) The following categories of candidates can be provisionally registered for the degree of Doctor of Philosophy in Mathematics.

a. Candidates having fellowships/scholarships instituted by the University/national and international agencies under schemes approved/recognized by the University.

b. Candidates who are otherwise eligible for admission to the Ph.D. Programme and do not have any financial assistance, will be awarded UGC (non-NET) Fellowship through an entrance examination to be conducted by the Department. The selection will be made on the basis of the relative merits considering 25% of marks obtained at the Undergraduate examinations, 25% of marks obtained at the Master's degree examinations, and 50% of marks scored in the entrance test. The minimum qualifying mark shall be 60%.

c. Foreign students with their national or other fellowships recognized by the University of sponsored by their employers, may be given provisional admission, followed by confirmation through due process after a stipulated period of time.

d. The University/College teachers holding a permanent, temporary or ad hoc positions and having completed two years of service as teacher in a Department/Constituent Colleges of the University of Delhi.

e. Candidates sponsored by their employers shall be considered only if they get study leave for a period of two years to fulfill residency requirements of the University of Delhi. Provided however in order to advance research in strategic areas of national concern, scientist/professionals working at defense and space institutions/organizations of the Government of India/State Government and with whom the University has signed a Memorandum of Under, will be allowed to purse Ph.D. while working in their organizations provided that the DRC recommends that their work in their parent organization is relevant to their Ph.D. research. Such students may also be exempt from the requirement of course work under ***amended Clause 4 E of Ordinance VI-B*** (amended vide notification dated 29th November, 2013)

f. Permanent teachers/ employees who are in service in any other recognized University/ College/ Research Institute in Indian and have a minimum of three years teaching/ research experience, will be considered if they get study leave for a period of two years to fulfill residency requirements of the University of Delhi.

2. REGISTRATION

A candidate for Ph.D. Programme in the Department of Mathematics will be registered by the Board of Research Studies (Mathematical Sciences) on the recommendations of the Departmental Research Studies subject to the availability of a supervisor in the area of his/her interest.

3. COURSE WORK

All candidates, except those who have been exempted from course work by the DRC, will be required to qualify Pre-Ph.D examination with three courses, selecting at most two from a group listed hereunder.

Group A

(i) Distribution Theory & Calculus on Banach Spaces
(iii) Operator Theory and Function Spaces
(iv) Geometric Function Theory
(v) Introduction to Operator Algebras
(vi) Advanced Frame Theory

Group B

(i) Rings and Modules
(ii) Differential Manifolds
(iii) Group Rings

Group C

(i) Graph and Network Theory
(ii) Convex and Non smooth Analysis
(iii) Combinatorial Mathematics
(vii) Advanced Compressible Flows

4. SCHEME OF EVALUATION

The candidates will be evaluated in each course out of 75 marks. The evaluation in each course will be based on the student's performance in a written examination and internal assessment. Each paper will be of 50 marks and of 3 hrs. duration. 25 marks in each course are assigned for internal assessment which will be based on assignments, attendance, class-room performance and seminars.

(i) 50% marks in each course will be required to pass.

(ii) Supplementary examination will be conducted for those who fail in the course work examination. Failed students can appear only in the supplementary examination of the same year which will be conducted within 3 months of the declaration of the result of course work.

(iii) No student shall be allowed to appear in course work examination more than twice.

5. "CRITERION FOR RECOGNIZING SUPERVISORS FROM COLLEGES OF UNIVERSITY OF DELHI:

(i) A permanent college teacher with at least 3 years of teaching experience and 2 years of research experience after obtaining Ph.D. degree may be recognized as a supervisor for guiding Ph.D. students.

(iv) For recognition, the teacher should be active in research; he/she should have published at least one research paper in journals listed in SCI/matscinet/scopus/any recognized Data Base from post-Ph.D. work during last five years.

(iii) The number candidates permitted to be registered may be as follows:

Associate Professor: 4 Assistant Professor: 2

iv) The above guidelines shall be applicable for new registrations even if the supervisors have already registered students.

6. FUNCTIONING OF ADVISORY COMMITTEE

i) Supervisors as convener of the Advisory Committee must ensure that a meeting of the committee takes place at least once in a year. The candidate may also be asked to make presentation before the committee. Minutes of the meetings should be submitted to the Department. This will be applicable to all registered students.

ii) All the annual reports of the candidates to be sent to BRS should go through DRC to enable DRC to verify if the meetings of the advisory committees have taken place.

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF DELHI
DELHI – 110007

Tel. No. 27666658

M.Phil (Full-time) Programme in Mathematics

(Revised in view of minutes of M. Phil. Meeting held on 8th April, 2013)

1. **ELIGIBILITY:** The candidate should have good academic record with first or high second class Master's Degree or an equivalent degree of a foreign University in the subject concerned, or an allied subject to be approved by the Vice-Chancellor on the recommendation of the Head of the Department and the Dean of the Faculty concerned.

2. **ADMISSION PROCEDURE:** Admission to the M.Phil Programme will be done on the basis of the relative merits of student's performance at Undergraduate and Post-graduate examinations and a written test to be conducted by the Department. The merit list will be prepared by taking into account 25% of marks scored in each of Undergraduate and Post-graduate examinations and 50% marks scored in the test. The M.Phil committee shall assign a Supervisor to each candidate and constitute an Advisory Committee of 3 members including the Supervisor of the candidate.

3. **PROGRAMME DESCRIPTION:** The M.Phil Programme shall consist of two parts:

Part-I (Course Work): A student must take four courses selecting from at least two different groups listed below.

Group- A

i) Distribution Theory and Calculus on Banach Spaces
ii) Matrix Analysis
iii) Operator Theory and Function Spaces
iv) Geometric Function Theory
v) Introduction to Operator Algebras
vi) Advanced Frame Theory

Group- B

i) Rings and Modules
ii) Group Rings
iii) Differential Manifolds
iv) Topological Structures

Group- C

i) Graph and Network Theory
ii) Convex and Non smooth Analysis
iii) Combinatorial Mathematics
iv) Parallel Iterative methods for Partial Differential Equations
v) Hydrodynamic Stability Theory
vi) Multi-objective Optimization
vii) Advanced Compressible Flows

The candidate will be examined for their coursework out of 300 marks and each course will carry 75 marks. Each course will be of 3 lecture/week and shall be expected to be completed in 40 lectures.

Part II (Dissertation)

(i) A candidate shall be required to write a dissertation under the guidance of a supervisor appointed by the M.Phil Committee. The dissertation will consist of a critical survey of some topic of interest in Mathematics, and /or involving research component.

(ii) Title of the dissertation should be approved and the appointment of external examiner be made before the submission of the dissertation. The application for approval of the title must include a synopsis together with a list of selected main references.

(iii) The candidate will be evaluated in Part-II examination out of 200 marks.

4. DURATION: The duration of the Programme will be one and half years. The dissertation can only be submitted afer one year from the date of admission to the M. Phil programme subject to qualifying Part–I examination by a candidate. However, a student must clear Part-I of M.Phil examination and submit a dissertation within three years of the initial registration for M.Phil Programme. No student shall be allowed to take up any assignment outside the University Department during the programme or before submission of his/her dissertation whichever is earlier.

5. ATTENDANCE

The minimum percentage of lectures to be attended and seminars to be participated in by the students shall be determined by the M. Phil. Committee of the Department. But in no case minimum requirement to be prescribed in any Department, shall be less than 2/3 of the lectures delivered and seminars held separately.

6. SCHEME OF EXAMINATION

(a) The Evaluation in each course will be based on the students performance in Written examination and internal assessment. The written examination of three hours duration will comprise of 50 marks. The internal assessment on the basis of assignment, attendance, class-room performance and seminars will comprise of 25 marks.

(b) Supplementary examination will be conducted for those who failed in the Part-I examination and it will be conducted within three months of the declaration of the result of Part-I examination. Students can appear in the supplementary examination only in the papers in which they fail. No student shall be allowed to appear in any course of the Part-I examination more than twice.

(c) No student shall be allowed to reappear in any course of Part-I examination just to improve upon the score.

(d) The dissertation shall be evaluated by the supervisor and one more examiner to be appointed by the M.Phil Committee.

(e) The total marks for Part – II examination is 200. The weightage of written dissertation will be 150 marks. Both the examiners will be required to submit marks out of 75 separately to the Head of the Department before fixing the date for viva-voce. The remaining 50 marks for the viva-voce shall be awarded jointly by both examiners.

(f) Dissertation can be resubmitted after revision if it is recommended so by the examiners. It cannot be submitted more than twice.

7. RESULT

a. 50% marks in each course will be required to pass the Part-I examination.

b. The examination result will be classified into the following three categories:

i. I Division With Distinction	At least 75% marks in the aggregate.
ii. I Division	At least 60% marks in the aggregate but below 75% marks.
iii. Pass	At least 50% marks separately in Part-I and Part-II.

PROGRAMME DESCRIPTION: The M.Phil Programme shall consist of two parts:

Part-I (Course Work): A student must take four courses selecting from at least two different groups listed below.

Group- A

i) Distribution Theory and Calculus on Banach Spaces
ii) Matrix Analysis
iii) Operator Theory and Function Spaces
iv) Geometric Function Theory
v) Introduction to Operator Algebras
vi) Advanced Frame Theory

Group- B

i) Rings and Modules
ii) Group Rings
iii) Differential Manifolds
iv) Topological Structures

Group- C

i) Graph and Network Theory
ii) Convex and Non smooth Analysis
iii) Combinatorial Mathematics
iv) Parallel Iterative methods for Partial Differential Equations
v) Hydrodynamic Stability Theory
vi) Multi-objective Optimization
vii) Advanced Compressible Flows

The candidate will be examined for their coursework out of 300 marks and each course will carry 75 marks. Each course will be of 3 lecture/week and shall be expected to be completed in 40 lectures.

<u>Group- A</u>

Distribution Theory and Calculus on Banach Spaces

Test functions and distributions, some operations with distributions, local properties of distributions, convolutions of distributions, tempered distributions and Fourier transform, fundamental solutions.

The Frechet derivative, chain rule and mean value theorems, implicit function theorem, extremum problems and Lagrange multipliers.

References:

[1] W. Cheney : Analysis for Applied Mathematics; Springer -Verlag, 2001.
[2] S. Kesavan : Topics in Functional Analysis and Applications; New Age International Publishers, 2008
[3] W. Rudin : Functional Analysis; Tata Mc-Graw Hill, 1991.
[4] Robert S. Strichartz : A guide to distribution theory and Fourier transforms; World Scientific Publishing Co., 2003.

Matrix Analysis

Unitary equivalence and normal matrices; Schur's Unitary triangularization theorem and its implications; QR-decomposition (factorization) canonical forms: The Jordan form and its applications; Other canonical forms and factorizations, Polar decomposition; Triangular factorizations, LU-decomposition, Norms for vectors and matrices; vector norms on matrices. Positive definite matrices; the Polar form and singular value decompositions. The Schur's product theorem; congruences; the positive definite ordering. Non-negative matrices and Primitive matrices. Stochastic and doubly stochastic matrices.

Contents are relevant sections of Ch. 2 to Ch.8 [2].

References:

[1] A. Bermann and R. Plemmans: Non-negative Matrices in Mathematical Sciences, Academic Press, 19+79.
[2] R.A. Horn and C.R. Johnson: Matrix Analysis, Vol. I, Cambridge Univ. Press, 1985.
[3] H. Minc: Non-negative matrices, Wiley Interscience, 1988.
[4] E. Seneta: Non-negative matrices, Wiley, New York, 1973.

Operator Theory and Function Spaces

Fredholm operators; semi-Fredholm operators; index of a Fredholm (semi- Fredholm) Operator; essential spectrum; Weyl spectrum and Weyl theorem; direct sums of operators, their spectra and numerical ranges; weighted shifts, their norms and spectral radii; normaloid, convexoid and spectraloid operators.

Invariant subspace problem; transitive, reductive and reflexive algebras; von-Neumann algebras.

Hardy spaces: Poisson's kernel; Fatou's theorem; zero sets of functions; multiplication, composition, Toeplitz and Hankel operators.

References

[1] Vladimir V.Peller, Hankel operators and their applications, Springer, 2002.
[2] Nikolai L.Vasilevski, Commutative algebras of Toeplitz operators on Bergman space, Birkhauser, 2008.
[3] N.Young, An introduction to Hilbert space, Cambridge University Press, 1988.
[4] P.R.Halmos, A Hilbert space problem book, II Ed., D.Van Nostrand Company, 1982.
[5] H.Radjavi and P.Rosenthal, Invariant subspaces, Springer Verlag, 1973.

Geometric Function Theory

Area theorem, growth, distortion theorems, coefficient estimates for univalent functions special classes of univalent functions. Lowner's theory and its applications; outline of de Banges proof of Bieberbach conjecture. Generalization of the area theorem, Grunsky inequalities, exponentiation of the Grunsky inequalities, Logarithmic coefficients. Subordination and Sharpened form of Schwarz Lemma

References

[1] P. Duren, Univalent Functions, Springer, New York, 1983
[2] A. W. Goodman, Univalent Functions I & II, Mariner, Florida, 1983
[3] Ch. Pommerenke, Univalent Functions, Van den Hoek and Ruprecht, Göttingen, 1975.
[4] M. Rosenblum, J. Rovnyak, Topics in Hardy Classes and Univalent Functions, Birkhauser Verlag, 1994
[5] D. J. Hallenbeck, T. H. MacGregor, Linear Problems and Convexity Techniques in Geometric Function Theory, Pitman Adv. Publ. Program, Boston-London-Melbourne,1984.
[6] I. Graham, G. Kohr, Geometric Function Theory in One and Higher Dimensions,Marcel Dekker, New York, 2003.

Introduction to Operator Algebras

Basic definitions and examples of Banach*-algebras, Spectrum of a Banach algebra element, L^1-algebras and Beurling algebras, Tensor products of Banach algebras, Multiplicative linear functional, The Gelfand representations, Fourier algebra, Functional calculus of in C*-algebras, Continuity and homomorphisms, Approximate identities in C*-algebras, Quotient algebras of C*-algebras, Representations and positive linear functional, Double Commutation Theorem, Enveloping von Neumann algebra of a C*-algebra, Tensor products of C*-algebras.

References:

[1] J.Diximier, C*-algebras, North-Holland Amersdem, 1977.

[2] R.V. Kadison and J.R.Ringrose, Fundamentals of the theory of operator algebras, Graduate studies in Mathematics, 15, AMS, Providence, 1997.

[3] E.Kaniuth, A course in commutative Banach algebras, Springer Verlag, 2008.

[4] M.Takesaki, Theory of Operator algebras, Springer Verlag, 2001.

Advanced Frame Theory

An overview on frames. B-Spline Symmetric B-Splines. Frames of translates. The canonical dual frame . Compactly supported generators. An application to sam-pling theory. Shift-Invariant Systems, Frame-properties of shift-invariant system, Representations of the frame operator. Gabor Frames in $L^2(R)$. Basic Gabor frame theory, Tight Gabor frames, The duals of a Gabor frame, Explicit construction of dual frame pairs, Popular Gabor conditions Representations of the Gabor frame operator and duality. Wavelet frames in $L^2(R)$.

References

[1] O. Christensen, Frames and bases (An introductory course), Birkhauser, Boston (2008).

[2] I. Daubechies, Ten Lectures on wavelets, SIAM, Philadelphia (1992).

[3] R. Young, A introduction to non-harmonic Fourier series, Academic Press, New York (revised rst edition 2001).

<u>Group- B</u>

Rings and Modules

Essential and superfluous submodules, Decomposition of rings, Generating and cogenerating, Modules with composition series, Fitting Lemma, Indecomposable decompositions of modules, Projective modules and generators, Radicals of projective modules, Projective covers, Injective hulls, Cogenerators, Flat modules. Singular submodules, Localization and maximal quotient rings. Essential finite generation, Finite dimensionality, Uniform modules and Goldie rings. Regular rings, Strongly regular rings, Unit regular rings, Right π- regular rings. Baer rings, Rickart rings. Baer*rings, Rickart*rings.

References:

[1] A.F.Anderson and K.R.Fuller: Rings and categories of modules, Springer- Verlag,1991 (Relevant sections of Ch. 2,3,4,5).

[2] S.K.Berberian : Baer Vings,Springer Verlag, New York ,1972 (Ch.1, sections 3, 4).

[3] K.R.Goodear1 : Ring theory (Non singular rings and modules), Marcel Dekker,Inc. New York (Relevant sections of Ch. 1,2,3).

[4] K.R.Goodear1 : Von Neumann regular rings,Pitman, London, 1979 (Ch. 1,3,4).

[5] T.Y.Lam: Lectures on Modules and rings, Springer Verlag, 1998(Ch. 3 ,section 7(d)).

Group Rings

Twisted Group Rings, Tensor Products, Idempotents, Finite groups, Augmentation annihilators, Group algebra as injective modules, Linear identities. The Center, Finite conjugate groups, Chain conditions.

References

[1] Donald S. Passman The Algebraic structure of Group Rings, , John Wiley and Sons, 1977.

[2] S.K. Sehgal, Topics in Group Rings, Marcel Dekker, New York, and Basel, 1978.

[3] , I.B.S. Passi, Group Rings and their augmentation idealsLecture Notes in Mathematics 715, Springer, New York, 1979.

[4] , A. A. BOVDI, Group RingsUzhgorod State University, 1978.

[5] D.S. Passman Infinite Group Rings, , Pure and Applied Math. 6, Marcel Dekkar, New York, 1971.

[6] Rings and Modules, P. Rihenboim, Interscience Tracts in Pure and Applied Mathematics, No.6, Interscience, New York, 1969.

Introduction to Differential Manifolds

The derivative, continuously differentiable functions, the inverse function theorem, the implicit function theorem. Topological manifolds, partitions of unity, imbeddings and immersions, manifolds with boundary, submanifolds. Tangent vectors and differentials, Sard's theorem and regular values, vector fields and flows, tangent bundles, smooth maps and their differentials. Smooth manifolds, smooth manifolds with boundary, smooth sub-manifolds, construction of smooth functions.

References:

[1] G.E. Bredon, Topology and Geometry, Springer-verlag, 1993.
[2] L. Conlon, Differentiable Manifolds, Second Edition, Birkhauser,2003.
[3] A. Kosinski, Differential Manifolds, Academic Press, 1992.
[4] M. Spivak, A Comprehensive Introduction to Differential Geometry, Vol. I; Publish or Perish, 1979.

Topological Structures

<u>Dimension Theory</u>: Definition and basic properties of the three dimension function inc. Inc. and dim, Characterization and subset theorems, equality of dim X and dim βX equality of Ind X and Ind βX.

<u>Paracompactness:</u> Paracompactness and full normality, presentation of paracompactness under mappings, Hanai-Moritastone theorem, products of paracompact spaces, countable paracompactness, strong paracompactness characterizations of strong paracompactness in regular spaces, products and subspaces of strongly paracompact spaces, pointwise paracompactness Arens Dugundji theorem, collectionwise normal spaces, Ding's example of a normal space which is not collectionwise normal.

<u>Bitopological Spaces</u>: Basic concepts, subspaces and products Separation and covering axioms.

References:

[1] R. Engelking: General Topology, Polish Scientific Publishers Warszawa, 2nd Ed., 1977.
[2] K. Nagami: DimensionTheory, Academic Press, New York, 1970.
[3] W.J. Pervin: Foundations of General Topology, Academic Press Inc., New York, 1964.
[4] S. Willard: General Topology, Addison-Wesley Publishing Co. Inc., 1970.

Group - C

Graph and Network Theory

Non-Oriented Linear Graphs: Introduction of graphs & networks, Paths & Circuits =, Euler Graph, M-Graph, Non-separable graph, Collection of Paths, Traversability: Eulerian Graphs & Hamiltonian Graphs.

Matrix Representation of Linear Graphs & Trees: Incidence Matrix, Tress, Spanning trees, Steiner Trees, Bottleneck Steiner trees, Forests, Branching, Circuits matrix.

Oriented Linear Graphs: Incident & Circuit matrices of Oriented graphs, Elementary tree transformation values of non zero major determinants of a circuit matrix.

Graphs Theory Algorithms, Dijikstra's Algorithm for finding the shortest path in a Network, Double Sweep Algorithm for finding k-shortest paths for a given k. Spanning tree Algorithm, Minimum Spanning Tree Algorithm-Maximum Branching Algorithm.

References:

[1] Mayeda W. : Graph Theory, Wiley-Interscience, John Wiley & Sons, Inc. 1972.
Harary F. : Graph Theory, & Theoretical Physics, Academic Press, 1967.
[2] Evans J.R. & Minicka E. : Optimization Algorithms for Networks & Graphs (2nd Edition) Marcel Deckar, 1992.
[3] V. Chachre, Ghere P.M. & Moore J.M.: Applications of Graph Theory Algorithm, Elsevier North Holland, Inc. 1979.
[4] Thulasiraman K. and Swami M.N. S – Grapha: Theory & Algorithms, Wiley Interscience Publication, 1992.

Convex and Nonsmooth Analysis

Convex sets, Convexity-preserving operations for a set, Relative Interior, Asymptotic cone, Separation theorems, Farkas Lemma, Conical approximations of convex sets, Bouligand tangent and normal cones. Convex functions of several variables, Affine functions, Functional operations preserving convexity of function, Infimal convolution, Convex hull and closed convex hull of a function, Continuity properties, Sublinear functions, Support function, Norms and their duals, Polarity. Subdifferential of convex functions, Geometric construction and interpretation, properties of subdifferential, Minimality conditions, Mean-value theorem, Calculus rules with subdifferentials, Subdifferential as a multifunction, monotonicity and continuity properties of the subdifferential, Subdifferential and limits of gradients.

References:

[1] Convex Analysis and Minimization Algorithms I, Jean-Baptiste Hiriart-Urmty and Claude Lemarechal, Springer- Verlag, Berlin, 1996.

[2] Convex Analysis and Nonlinear Optimization : Theory and Examples, Jonthan M. Bonvein and Adrain S. Lewis, CMS Books in Mathematics, Springer Verlag, New York, 2006.

[3] Convex Analysis, R. Tyrrell Rockafellar, Priceton University Press, Princeton, New Jersey, 1997.

Combinatorial Mathematics

Permutations and combinations, The Rules of Sum and Product, Distributions of Distinct Objects, Distributions of Nondistinct Objects.

Generating Functions for Combinations, Enumerators for Permutations, Distributions of Distinct Objects into Nondistubct Cells, Partitions of Integers, Elementary Relations.

Recurrence Relations, Linear Recurrence Relations with Constant Coefficients, Solution by the technique of Generating Functions, Recurrence relations with two indices.

The Principle of Inclusion and Exclusion. The General Formula, Derangements, Permutations with Restrictions on Relative positions.

Polya's Theory of Counting, Equivalence Classes under a Permutation Group, Equivalence Classes of Functions, Weights and Inventories of Functions, Polya's Fundamental Theorem. Generalization of Polya's Theorem.

Block designs, Complete block designs, Orthogonal Latin Squares, Balanced Incomplete Block designs. Construction of Block designs.

References:

[1] Introduction to Combinatorial Mathematics by C.L. Ltd (McGraw-Hill), 1968.

[2] An Introduction to Combinatorial Analysis by J. Riordan (John Wiley & Sons), 1958.

[3] R P Grimaldi, Discrete and Combinatorial Mathematics, 4ed, Addision-Wesley, New York, 1998.

[4] S. Barnett, Discrete Mathematics, Numbers and Beyond, Addision-Wesley, Singapore, 1998

Parallel Iterative methods for Partial Differential Equations

Speedup; efficiency; Amdahl's law; point and block parallel relaxation algorithms (Jacobi, Gauss-Seidel, SOR); triangular matrix decomposition; quadrant interlocking factorisation method; red-black ordering; application to elliptic BVPs; parallel ADI algorithms; parallel conjugate-gradient method; parallel multi-grid method; parallel domain decomposition method.

The alternating group explicit method for two point BVPs (natural, derivative, mixed, periodic) and their convergence analysis; the MAGE and NAGE methods; the computational complexity of the AGE method; the Newton-AGE method.

Parabolic equation: AGE algorithm for diffusion-convection equation and its convergence analysis; stability analysis of more general scheme; CAGE method; AGE method for fourth order parabolic equation.

Hyperbolic equation: Group explicit method for first and second order hyperbolic equations; GER, GEL, GAGE, GEU, GEC algorithms; stability analysis of GE method; AGE iterative method for first and second order hyperbolic equations.

Elliptic equation: Douglas-Rachford algorithm; BLAGE iterative algorithm with different boundary conditions; AGE-DG algorithm; parallel implementation.

This course consists of theory paper and computer practical.

References:

[1] Y. Saad, Iterative Methods for Sparse Linear Systems, SIAM, Philadelphia (2003).

[2] L.A. Hageman and D.M. Young, Applied Iterative Methods, Dover publication, New York (2004).

[3] D.M. Young, Iterative Solution of Large Linear Systems, Academic Press, New York (1971).

[4] Jianping Zhu, Solving Partial Differential Equations on Parallel Computers, World Scientific, New Jersey (1994).

[5] D.J. Evans, Group Explicit Methods for the Numerical Solution of Partial Differential Equations, Gordon and Breach Science publisher, Amsterdam (1997).

Hydrodynamic Stability Theory

The concept of hydrodynamic stability, the stability of superposed fluids; the Rayleign-taylor instability-the case of two uniform fluids of constant densities separated by a horizontal boundary, the case of exponentially varying density. The Kelvin – Helmholtz stability.

The stability of coquette flow – RayIeigh's criterian. Analytical discussion of stability of inviscid Couette flow. Oscillations of a rotating column of liquid. Thermal stability Orr-Sommerfeld equation, Rayleigh's theorems.

References:

[1] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Oxford University Press, (1961)
[2] P.G. Draxin and W.H. Reid, Hydodynamic Stability, Cambridge University Press 1981)

Multi-objective Optimization

Multiple Objective Linear Programming Problem, Multiple Criteria Examples, Utility Functions,Non Dominated Criterion Vectors and Efficient Points, Point Estimate Weighted Sums Approach, Optimal Weighting Vectors, Scaling and Reduced Feasible Region Methods, Vector Maximum Algorithm. Formulation of the Multiple Objective Model, Method of Solutions, Augmented Goal Programming, Interactive Multiple Objective Methods. Multiple Objective Linear Fractional Programming. Multiple Objective Non linear Programming Problem, Efficiency and Non- Dominance, Weakly and Strictly Efficient Solutions, Proper Efficiency and Proper Non- Dominance. Weighted Sum Scalarization : (Weak) Efficiency, Proper Efficiency, Optimality Conditions. Scalarization Techniques : The €-Constraint Method, The Hybrid Method, The Elastic Constraint Method and Benson's Method.

References:

[1] Ralph E.Steuer : Multi-Criteria Optimization, Theory Computation and Application, John Wiley and Sons, 1986. Chapters-1, 6, 7, 8, 9, 12.
[2] James P.lgnizeo : Linear Programming in Single and Multiple Objective Systems, Prentice Hall Inc. , Englewood Cliffs, N.J - 07632, 1982. Chapters- 16, 17, 20.
[3] Matthias Ehrgott: Multicriteria Optimization, Springer Berlin. Heidelberg-2005, Second Edition, Chapters- 2, 3,4.

Advanced Compressible Flow

One-dimensional gas flow (with perfect and van der Waals gas, gravitation, viscosity, heat addition, and conduction), Diffusion, Shock waves (discontinuity surface, jump condition, strength, thickness, reflection, structure, heat addition and MHD effects), Detonation and Deflagration waves, Methods of solution of compressible flow problems. Dimensional analysis and similarity method, Self-similar motion of spherical, cylindrical and plane waves in a gas. Two dimensional subsonic and supersonic flow with linearized theory, Two dimensional subsonic potential flows(Rayleigh-Janzen Method), Two dimensional supersonic flow with method of characteristics. Anisentropic rotational flow of inviscid compressible fluid.

References:

1. Similarity and Dimensional Method in Mechanics, L.I.Sedov, Mir Publisher, 1982
2. Fluid Mechanics, L.D.Landau and E.M.Lifshitz, Pragamon Press, 1989
3. Introduction to the theory of compressible flow, S.I.Pai, D. Van Nostrand Company, 1958
4. Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Ya. B. Zel'dovich, and Yu.P. Raizer, Academic Press, 1966.
5. Introductory Fluid Mechanics, J.Katz , Cambridge University Press, 2010

UNIVERSITY OF DELHI

M.A./M.SC. MATHEMATICS

TWO-YEAR FULL TIME PROGRAMME

RULES, REGULATIONS AND COURSE CONTENTS

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF DELHI
DELHI-110007

July 2014

University of Delhi
Examination Branch

Check List of Course Evaluation for AC Consideration

S. No.	Parameter	Status
1	Affiliation	√
2	Programme Structure	√
3	Codification of Papers	√
4	Scheme of Examinations	√
5	Reappearance in Passed papers	√
6	Division Criteria	√
7	Qualifying Papers	√
8	Span Period	√
9	Attendance Requirements	×
10	Course Content for each paper	√
11	List of Readings	√

MASTER OF ARTS/SCIENCE
(MATHEMATICS)
TWO-YEAR FULL-TIME PROGRAMME

AFFILIATION

The proposed programme shall be governed by the Department of Mathematics, Faculty of Mathematical Sciences, University of Delhi, Delhi-110007.

PROGRAMME STRUCTURE

The master's programme in Mathematics is divided into two parts as hereunder. Each part will consist of two semesters.

Part I	First Year	Semester 1	Semester 2
Part II	Second Year	Semester 3	Semester 4

The courses prescribed for various semesters shall be the following:

Semester I	
MATH14-101	FIELD THEORY
MATH14-102	COMPLEX ANALYSIS
MATH14-103	MEASURE AND INTEGRATION
MATH14-104	DIFFERENTIAL EQUATIONS
Semester II	
MATH14-201	MODULE THEORY
MATH14-202	TOPOLOGY-I
MATH14-203	FUNCTIONAL ANALYSIS
MATH14-204	FLUID DYNAMICS
Semester III	
	Anyone of the following
MATH14-301(A)	ALGEBRAIC TOPOLOGY
MATH14-301(B)	REPRESENTATION OF FINITE GROUPS
MATH14-301(C)	COMMUTATIVE ALGEBRA
	Anyone of the following
MATH14-302(A)	FOURIER ANALYSIS
MATH14-302(B)	MATRIX ANALYSIS
MATH14-302(C)	THEORY OF BOUNDED OPERATORS
	Anyone of the following
MATH14-303(A)	ADVANCED COMPLEX ANALYSIS
MATH14-303(B)	MEASURE THEORY
MATH14-303(C)	TOPOLOGY-II

	Anyone of the following
MATH14-304(A)	CODING THEORY
MATH14-304(B)	COMPUTATIONAL FLUID DYNAMICS
MATH14-304(C)	COMPUTATIONAL METHODS FOR ODES
MATH14-304(D)	MATHEMATICAL PROGRAMMING
MATH14-304(E)	METHODS OF APPLIED MATHEMATICS
MATH14-304(F)	GRAPH THEORY
Semester IV	
	Anyone of the following
MATH14-401(A)	ALGEBRAIC NUMBER THEORY
MATH14-401(B)	THEORY OF NON-COMMUTATIVE RINGS
MATH14-401(C)	SIMPLICIAL HOMOLOGY THEORY
	Anyone of the following
MATH14-402(A)	ABSTRACT HARMONIC ANALYSIS
MATH14-402(B)	FRAMES AND WAVELETS
MATH14-402(C)	OPERATORS ON HARDY-HILBERT SPACES
MATH14-402(D)	THEORY OF UNBOUNDED OPERATORS
	Anyone of the following
MATH14-403(A)	CALCULUS ON $\mathbb{R}^n$
MATH14-403(B)	DIFFERENTIAL GEOMETRY
MATH14-403(C)	TOPOLOGICAL DYNAMICS
	Anyone of the following
MATH14-404(A)	ADVANCED CODING THEORY
MATH14-404(B)	ADVANCED FLUID DYNAMICS
MATH14-404(C)	COMPUTATIONAL METHODS FOR PDES
MATH14-404(D)	CRYPTOGRAPHY
MATH14-404(E)	DYNAMICAL SYSTEM
MATH14-404(F)	OPTIMIZATION TECHNIQUE AND CONTROL THEORY

SCHEME OF EXAMINATIONS

1. English shall be the medium of instruction and examination.

2. Examinations shall be conducted at the end of each Semester as per the Academic Calendar notified by the University of Delhi.

3. Each course will carry 100 marks and have two components: Internal Assessment 30% marks and End-Semester Examination 70% marks.

4. The system of evaluation shall be as follows:

4.1 Internal assessment will be based on classroom participation, seminar, term courses, tests, quizzes. The weightage given to each of these components shall be decided and announced at the beginning of the semester by the individual teacher responsible for the course. No special classes will be conducted for a student during other semesters, who fails to participate in classes, seminars, term courses, tests, quizzes and laboratory work.

4.2 The remaining 70 marks in each paper shall be awarded on the basis of a written examination at the end of each semester. The duration of written examination for each paper shall be three hours.

5. Examinations for courses shall be conducted only in the respective odd and even Semesters as per the Scheme of Examinations. Regular as well as Ex-Students shall be permitted to appear/re-appear/improve in courses of Odd Semesters only at the end of Odd Semester and courses of Even Semesters only at the end of Even Semesters.

PASS PERCENTAGE & PROMOTION CRITERIA:

(a) The minimum marks required to pass any paper in a semester shall be 40% in theory and 40% in Practical, wherever applicable. The student must secure 40% in the End Semester Examination and 40% in the total of End Semester Examination & Internal Assessment of the paper for both theory & practical separately.

(b) No student will be detained in I or III Semester on the basis of his/her performance in I or III Semester examination; i.e. the student will be promoted automatically from I to II and III to IV Semester.

(c) A student shall be eligible for promotion from Ist year to 2nd year of the course provided he/she has passed 50% papers of I and II Semester taken together. However, he/she will have to clear the remaining paper/s while studying in the 2nd year of the programme.

(d) Students who do not fulfill the promotion criteria (c) above shall be declared fail in the Part concerned. However, they shall have the option to retain the marks in the papers in which they have secured Pass marks as per Clause (a) above.

(e) A student who has to reappear in a paper prescribed for Semester I/III may do so only in the odd Semester examinations to be held in November/December. A student who has to reappear in a paper prescribed for Semester II/IV may do so only in the even Semester examinations to be held in April/May.

REAPPEARANCE IN PASSED PAPERS

(a) A student may reappear in any theory paper prescribed for a semester, on foregoing in writing her/his previous performance in the paper/s concerned. This can be done once only in the immediate subsequent semester examination only (for example, a student reappearing in a paper prescribed for Semester I examination, may do so along with the immediate next Semester III examinations only)

(b) A candidate who has cleared the papers of Part II (III & IV Semesters) may reappear in any paper of III or IV Semester only once, at the immediate subsequent examination on foregoing in writing her/his previous performance in the paper/s concerned, within the prescribed span period.

(Note: The candidate of this category will not be eligible to join any higher course of study)

(c) In the case of reappearance in a paper the result will be prepared on the basis of candidates current performance in the examination.

(d) In the case of a candidate, who opts to re-appear in any paper/s under the aforesaid provisions, on surrendering her/his earlier performance but fails to re-appear in the paper/s concerned, the marks previously secured by the candidate in the paper/s in which he/she failed to re-appear shall be taken into account while determining her/his result of the examination held currently.

(e) Reappearance in Practical examinations, dissertation, project and filed work shall not be allowed.

(f) A student who reappears in a paper shall carry forward the internal assessment marks, originally awarded.

NOTES: (1) Each course will have 5 credits: 4 lectures, 1 discussion and 1 tutorial per week.

(2) In the beginning of the respective semesters, the Department will announce the list of elective courses which will be offered during the semester depending upon the availability of lecturers and the demand of electives.

DIVISION CRITERIA

A student who passes all the papers prescribed for Part I & II examinations would be eligible for the degree. Such a student shall be categorized on the basis of the combined result of Part I and II examinations as follows:-

(1) 60% or more : Ist Division
(2) 50% or more but less than 60% : IInd Division
(3) 40% or more but less than 50% : IIIrd Division

SPAN PERIOD No student shall be admitted as a candidate for the examination for

any of the Parts/Semesters after the lapse of four years from the date of admission to the Semester 1 of the masters programme in Mathematics.

Semester I

MATH14-101: Field Theory

Fields and their extensions, splitting fields, the algebraic closure of a field, separability, automorphisms of field extensions, the fundamental theorem of Galois theory, roots of unity, finite fields, primitive elements, Galois theory of equations, the solution of equations by radicals.

Reference books.

[1] P.M. Cohn, Basic Algebra, Springer International Edition, 2003.
[2] P.M. Cohn, Classic Algebra, John Wiley & Sons Ltd., 2000.
[3] N. Jacobson, Basic Algebra I & II, Hindustan Publishing Co., 1989.
[4] T. W. Hungerford, Algebra, Springer-Verlag, 1981.

MATH14-102: Complex Analysis

Analytic functions as mappings, conformal mappings, Mobius transformations, branch of logarithm, Riemann Stieltjes integrals.

Power series representation of analytic functions, maximum modulus theorem, index of a closed curve,Cauchy's theorem and integral formula on open subsets of $\mathbb{C}$.

Homotopy, homotopic version of Cauchy's theorem, simple connectedness, counting of zeros, open mapping theorem, Goursat's theorem, Classification of singularities, Laurent series.

Residue, Contour integration, argument principle, Rouche's theorem, Maximum principle, Schwarz' lemma.

Text book(s).

[1] J. B. Conway, *Functions of One Complex Variable*, Narosa, New Delhi, 2002.

Reference books.

[1] L.V. Ahlfors, *Complex Analysis*, Mc. Graw Hill Co., New York, 1988.
[2] T. W. Gamelin, *Complex Analysis*, Springer Verlag, 2008.
[3] L. Hahn, B. Epstein, *Classical Complex Analysis*, Jones and Bartlett, India, New Delhi, 2011.
[4] D. Ullrich, *Complex Made Simple*, Amer. Math. Soc., 2008.

MATH14-103: Measure and Integration

Lebesgue outer measure, measurable sets, regularity, measurable functions, Borel and Lebesgue measurability, non-measurable sets

Integration of nonnegative functions, the general integral, integration of series, Riemann and Lebesgue integrals

Functions of bounded variation, Lebesgue differentiation theorem, differentiation and integration, absolute continuity of functions, Measures and outer measures, measure spaces, integration with respect to a measure

The L^p spaces, Holder and Minkowski inequalities, completeness of L^p spaces, convergence in measure, almost uniform convergence, Egorov's theorem.

Text book(s).

[1] G. de Barra, Measure and Integration. New Age International (P) Ltd., New Delhi, 2006.

Reference books.

[1] M. Capinski and E. Kopp, Measure, Integral and Probability, Springer-Verlag, 2003

[2] E. Hewitt and K. Stromberg, Real and Abstract Analysis, Springer, Berlin, 1988

[3] H. L. Royden, Real Analysis, 3rd Edition, Prentice Hall, 1988

MATH14-104 Differential Equations

Well posed problems. Existence, uniqueness and continuity of the solution of ordinary differential equation of first order, Picard's method. Existence and uniqueness of the solution of simultaneous differential equations of first order and ordinary differential equation of higher order. Sturm separation and comparison theorems, Homogeneous linear systems, Non-homogeneous Linear systems, Linear systems with constant coefficients.

Two point boundary value problems, Greens function, Construction of Green functions, Sturm-Lioville systems, Eigen values and Eigen functions. Stability of autonomous system of differential equations, critical point of an autonomous system and their classification as stable, asymptotically stable, strictly stable and unstable. Stability of linear systems with constant coefficients. Linear plane autonomous systems, Perturbed systems. Method of Lyapunov for nonlinear systems.

Fourier transform and its application to solution of PDEs, Boundary value problems, Maximum and minimum principles, Uniqueness and continuous dependence on boundary data, Solution of the Dirichlet and Neumann problem for a half plane by Fourier transform method. Solution of Dirichlet problem for a circle in form of Poission integral formula. Theory of Green function for Laplace equation in two dimension and its application in solution of Dirichlet and Neumann problem for half plane and circle, Theory of Green function for Laplace equation in three dimension and its application in solution of Dirichlet and Neumann Problem for semi-infinite space and sphere.

Wave equation, Helmholtz's first and second theorems, Green's function for wave equation. Duhamel's principles for wave equation, Diffusion equation, Solution of initial boundary value problems for the diffusion equation, Green's function for diffusion equation, Duhamel's principles for heat equation.

Text book(s).

[1] E.A. Coddington, An Introduction to Ordinary Differential Equations, Dover, 1989

[2] Tyn Myint-U, Ordinary Differential Equations, Elsevier North-Holland, 1978

[3] Ian N. Sneddon, Elements of Partial Differential Equations, McGraw-Hill, 1986

Reference books.

[1] G.F. Simmons, Ordinary Differential Equations with applications and Historical notes, McGraw-Hill, 1991.

[2] Tyn Myint-U Linear Partial Differential Equations for Scientists and Engineers, Birkhauser, 2007.

[3] S.L.Ross, Differential Equation, Wiley India, 2004

Semester II

MATH14-201: Module Theory

Modules, Basic Concepts, Direct product and Direct sums, Exact sequences, Split exact sequences, Nakayama lemma, Free modules, Modules over P.I.D., Chain conditions, Hilbert basis theorem, Categories and Functors, Hom functors, Tensor product of modules, Semi simple modules, Projective and Injective modules, Baer's criterion, Divisible modules.

Reference books.

[1] P.M. Cohn, Classic Algebra, John Wiley & Sons Ltd., 2000.
[2] P.M. Cohn, Basic Algebra, Springer International Edition, 2003.
[3] D. S. Dummit & R.M. Foote, Abstract Algebra, Wiley India Pvt. Ltd.
[4] T.W. Hungerford, Algebra, Springer-Verlag, 1981.
[5] N. Jacobson, Basic Algebra, Volume II, Hindustan Publishing Co., 1989.

MATH15-202: Topology - I

Topological spaces, derived concepts: interior, closure, boundary and limit points of subsets, basis and subbasis for a topology, order topology, subspaces, continuous functions, homeomorphism, product topology, metrisability of products of metric spaces, connected spaces, components, path connected spaces, local connectedness, local path-connectedness, convergence: sequences and nets, Hausdorff spaces, 1st and 2nd countable spaces, separable and Lindelöf spaces, compactness, Tychonoff Theorem, Bolzano-Weierstrass property, countable compactness.

Reference books.

[1] G. E. Bredon, Topology and Geometry, Springer-Verlag, 2005.
[2] J. Dugundji, Topology, Allyn and Bacon, 1970.
[3] J.L. Kelley, General Topology, Springer-Verlag, 2005.
[4] J. R. Munkres, Topology, second edition, Pearson Education, 2003.
[5] T. B. Singh, Elements of Topology, CRC Press, 2013.
[6] S. Willard, General Topology, Dover Publications, Inc. N.Y., 2004.

MATH14-203: Functional Analysis

Normed spaces, Banach spaces, finite dimensional normed spaces and subspaces, compactness and finite dimension. Bounded and continuous linear operators, linear operators and functionals on finite dimensional spaces. Normed spaces of operators, dual space.

Hilbert spaces, orthogonal complements and direct sums, Bessel inequality, total orthonormal sets and sequences. Representation of functionals on Hilbert spaces. Hilbert adjoint operator. Self-adjoint, unitary and normal operator.

Hahn Banach theorems for real / complex and normed spaces. Adjoint operator, reflexive spaces. Uniform boundedness theorem strong and weak convergence, convergence of sequences of operators and functionals. Open mapping theorem, closed graph theorem

Spectrum of an operator, spectral properties of bounded linear operators, non-emptiness of the spectrum.

Text book(s).

[1] E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley and Sons (Asia) (2011).

Reference books.

[1] G. Bachman and L. Narici, Functional Analysis, Dover Publication, N.Y. (2000).
[2] R. Bhatia, Notes on Functional Analysis, Hindustan Book Agency (India)(2009).
[3] M. Schechter, Principles of Functional Analysis, AMS, 2002.

Math14-204 Fluid Dynamics

Classification of fluids, the continuum model, Eulerian and Lagrangian approach of description. Differentiation following fluid motion. Irrotational flow,vorticity vector, equi-potential surfaces. Streamlines, pathlines, streak lines of the particles, stream tube and stream surface. Mass flux density, conservation of mass leading to equation of continuity. (Euler's form.) Conservation of momentum and its mathematical formulation: Euler's form. Integration of Euler's equation under different conditions. Bernoulli's equation, steady motion under conservative body forces,

Boundary surface, Theory of irrotational motion, Kelvin's minimum energy and circulation theorems, potential theorems. Some two-dimensional flows of irrotational, incompressible fluids. Complex potential. Sources, sinks, doublets and vortices. Milne-Thomson circle theorem, Images with respect to a plane and circles. Blasius theorem.

Three-dimensional flows. Sources, sinks, doublets. Axi-symmetric flow and Stokes stream function. Butler sphere theorem, Kelvin's inversion theorem and Weiss's sphere theorem. Images with respect to a plane and sphere. Axi-symmetric flows and stream function. Motion of cylinders and spheres.

Viscous flow, stress and strain analysis. Stokes hypothesis, The Navier-Stokes equations of motion. Some exactly solvable problems in viscous flows, steady flow between parallel plates, Poiseuille flow, steady flow between concentric rotating cylinders.

Text book(s).

[1] F.Chorlton : Text book of Fluid Dynamics , CBS 2004.

Reference books.

[1] P.K. Kundu and I.M. Cohen, Fluid Mechanics, Academic Press, 2005.
[2] L.M.MilneThomson, Theoretical Hydrodynamics, The Macmillan company, USA, 1969.
[3] N.E.Neill and F. Chorlton, Ideal and incompressible fluid dynamics, Ellis Horwood Ltd, 1986.
[4] N.E.Neill and F. Chorlton, Viscous and compressible fluid dynamics, Ellis Horwood Ltd, 1986.
[5] D.E.Rutherford: Fluid Dynamics, Oliver and Boyd Ltd, London, 1978.

Semester III

MATH14-301(a): Algebraic Topology

Homotopic maps, homotopy type, retract and deformation retract. Fundamental group. Calculation of fundamental groups of n-sphere, the cylinder, the torus, and the punctured plane. Brouwer fixed-point theorem, the fundamental theorem of algebra, free products, free groups, Seifert-Van Kampen theorem and its applications. Covering projections, the lifting theorems, relations with the fundamental group, universal covering space. The Borsuk-Ulam theorem, classification of covering spaces.

Reference books.

[1] G.E. Bredon, Geometry and Topology, Springer-Verlag, 2005.
[2] W.S. Massey, A Basic Course in Algebraic Topology, Springer, 1991.
[3] J.J. Rotman, An Introduction to Algebraic Topology, Springer-Verlag, 2004.
[4] T. B. Singh, Elements of Topology, CRC Press, 2013.
[5] E.H. Spanier, Algebraic Topology, Springer-Verlag, 1989.

MATH14-301(b): Representation of Finite Groups

Representation of groups, right regular representation, coset representation, matrix representation, linear representation, trivial representation, equivalent matrix representations, G-modules, automorphism representations, characters, class function, reducibility, reducible and irreducible G-modules, contra gradient representation, permutation representations, complete irreducibility, Maschke's theorem for matrix representations and G-modules, Schur's lemma for matrix representations and G-modules, commutant (endomorphism) algebra.

Elementary property of group characters, orthogonality relations, inner product for functions on a group G, orthogonal functions, character relations of the first kind, simple and compound characters, group algebra, character table, character relatioins of the second kind, character table for finite abelian groups, the lifting process, linear characters.

Induced representations, induced characters, restricted character, reciprocity theorem of Frobenius, character tables for alternating groups of degree 4 and 5, conjugate characters, Clifford's theorem, tensor products and Mackey's theorem, Algebraic numbers and conjugates, algebraic integers and their properties, representation of group algebras, Burnside's (p,q)-theorem, Frobenius groups.

Text book(s).

[1] James Gordan and Martin Lieback, Representations and characters of groups, Cambridge University Press, Cambridge, 2001.

Reference books.

[1] Charles W. Curtis and Irving Reiner, Representation Theory of finite groups and associative algebras, AMS Chelsea Publishing, American Mathematical Society reprint, 2006.

[2] William Fulton and Joe Harris, Representation Theory: A first course, Springer-Verlag, New York Inc., 1991.
[3] I. Martin Isaacs, Character Theory of finite groups, AMS Chelsea Publishing, American Mathematical Society reprint, 2006.
[4] Walter Ledermann, Introduction to group characters, Cambridge University Press, Cambridge, 1987.
[5] J. P. Serre, Linear representation of finite groups, Springer-Verlag, 1977.

MATH14-301(c): Commutative Algebra

Extension and Contraction of ideals, Prime spectrum of Rings, Jacobson radical of a ring, Prime avoidance lemma, Rings of formal power series, Restriction and extension of scalars.

Localisation, Local properties, Extended & contracted ideals in rings of fractions, Primary decomposition, First and second uniqueness theorem of primary decomposition, Noetherian rings, Primary decomposition in Noetherian rings, Artin rings, Structure theorem for Artin rings.

Integral dependence, Going up theorem, Going down theorem, Integrally closed domains, Valuation rings, Hilbert's Nullstellensatz theorem, Discrete valuation rings, Dedekind domains, Fractional ideals.

Text book(s).

[1] M.F. Athiyah & I.G. Macdonald, Introduction to Commutative Algebra, Addison Wesley, 1969.

Reference books.

[1] Balwant Singh, Basic Commutative Algebra, World Scientific Publishing Co., 2011.
[2] D. Eisenbud, Commutative Algebra with a view towards algebraic geometry, Springer Verlag, 1995.
[3] O. Zariski & P. Samuel, Commutative Algebra, Vol. 1 & 2, Springer-Verlag, 1975.
[4] R.Y. Sharp, Steps in Commutative Algebra, Cambridge University Press, 1990

MATH14-302(a): Fourier Analysis

Convergence and divergence of Fourier series, Fejer's theorem, approximate indentities, the classical kernels [Fejer's, Poisson's and Dirichlet's summability in norm and pointwise summability], Fatou's theorem.

The inequalities of Hausdorff and Young, examples of conjugate function series, the Fourier tranform, kernels on $\mathbb{R}$.

Basic properties of topological groups, separation properties, subgroups, quotient groups and connected groups, Notion of Haar measure on topological groups with emphasis on $\mathbb{R}, \mathbb{T}$ and $\mathbb{Z}$ and some simple matrix groups, $L^1(G)$ and convolution with special emphasis on $L^1(\mathbb{R})$, $L^1(\mathbb{T})$ and $L^1(\mathbb{Z})$.

Plancherel theorem on abelian groups, Plancherel measure on $\mathbb{R}, \mathbb{T}$ and $\mathbb{Z}$, maximal ideal space of $L^1(G)$ (G an abelian topological group).

Text book(s).

[1] Y. Katznelson, Introduction to Harmonic Analysis, John Wiley, 2004.

Reference books.

[1] H. Helson, Harmonic Analysis, Addison-Wesley, 983, Hindustan Pub. Co., 1994.
[2] E. Hewitt and K.A. Ross, Abstract Harmonic Analysis, Vol I, Springer-Verlag, 1993

MATH14-302(b): Matrix Analysis

Closed subgroups of general linear group. Examples and their compactness and connectedness. Matrix exponential.

Norm for vectors and matrices. Analytic properties of vector norms. Geometric properties of vector norms. Matrix norms. Error in inverses and solution of linear systems.

Location and perturbation of eigenvalues. Geršgorin discs, other inclusion regions. Positive definite matrices.

Polar form and singular value decomposition. The Schur product theorem. Positive semi-definite ordering. Inequalities for positive definite matrices. Majorisation and doubly stochastic matrices.

Text book(s).

[1] R. A. Horn and C. R. Johnson, Matrix Analysis, Cambridge University Press (2010)
[2] B. C. Hall, Lie Groups, Lie Algebras, and Representations: An Elementary Introduction, Springer-Verlag (2003)

Reference books.

[1] R. Bhatia, Matrix Analysis Springer-Verlag (1996)
[2] C. D. Meyer, Matrix Analysis and Applied Linear Algebra, SIAM (2000).
[3] F. Zhang, Matrix Theory (Basic Results and Techniques), Springer-Verlag (1999).

MATH14-302(c): Theory of Bounded Operators

Spectrum of a bounded operator: Review of basic concepts, point, continuous and residue spectrum, and of notions of uniform, strong and weak operator convergence on the space of bounded linear operators. Approximate point spectrum and compression spectrum, spectral mapping theorem for polynomials.

Compact linear operators: Basic properties, adjoint of compact operators, Spectral properties of compact operators, the Fredholm alternative.

Spectral theory of self-adjoint operators : spectral properties of self adjoint operators, positive operators and their properties, spectral representation of a self adjoint compact operator, spectral family of a self adjoint operator and its properties, spectral representation of a self adjoint operator, continuous functions of self-adjoint operators.

Polar decomposition, singular values, trace class operators, trace norm and trace, Hilbert Schmidt operators.

Reference books.

[1] R. Bhatia, Notes on Functional Analysis, TRIM series, Hindustan Book Agency, India, 2009.
[2] J.E. Conway, A course in Operator Theory, Graduate Studies in Mathematics, Volume 21, AMS (1999)
[3] E. Kreyszig, Introductory Functional Analysis with Applications,John Wiley and Sons (2001)
[4] Martin Schechter, Principles of Functional Analysis, American Mathematical Society, (2004)

MATH14-303(A): Advanced Complex Analysis

Hadamard's three circles theorem, Phragmen-Lindelof theorem. The space of continuous functions $C(G, \Omega)$, spaces of analytic functions, Hurwitz's thorem, Montel's theorem, spaces of meromorphic functions.

Riemann mapping theorem, Weiersirass' factorization theorem, factorization of the sine function. Runge's theorem, simply connected regions, Mittag-Leffler's theorem

Harmonic functions, maximum and minimum principles, harmonic functions on a disk, Harnack's theorem, sub-harmonic and super-harmonic functions, maximum and minimum principles, Dirichlet problem, Green's function.

Entire functions. Jensen's formula, Bloch's theorem, Picard theorems, Schottky's theorem.

Text book(s).

[1] J. B. Conway, *Functions of One Complex Variables*, 2nd ed, Narosa Publishing House, New Delhi, 2002.

Reference books.

[1] L.V. Ahlfors, *Complex Analysis*, Mc. Graw Hill Co., New York, 1988.
[2] L. Hahn, B. Epstein, *Classical Complex Analysis*, Jones and Bartlett, India, New Delhi, 2011.
[3] W. Rudin, *Real and Complex Analysis*, McGraw-Hill, 1987
[4] D. Ullrich, *Complex Made Simple*, Amer. Math. Soc., 2008

MATH14-303(B): Measure Theory

Signed measures, complex measures, Hahn decomposition theorem, Jordan decomposition theorem, mutually singular measures, Radon-Nikodym theorem, Lebesgue decomposition.

Caratheodory extension theorem, Lebesgue measure on $\mathbb{R}^n$, uniqueness up to multiplication by a scalar of Lebesgue measure in $\mathbb{R}^n$ as a translation invariant Borel meaasure.

Riesz representation theorem for bounded linear functionals on L^p-spaces, Product measures, Fubini's theorem, Tonelli's theorem.

Baire sets, Baire measures, continuous functions with compact support, regularity of measures on locally compact spaces, Regularity of Lebesgue measure in $\mathbb{R}^n$. Riesz Markov representation theorem.

Text book(s).

[1] H. L. Royden, Real Analysis, 3rd Edition, Prentice Hall, 1988

Reference books.

[1] C. D. Aliprantis and O. Burkinshaw, Principles of Real Analysis, Academic Press, Indian Reprint, 2011
[2] A. K. Berberian, Measure and Integration, AMS Reprint, 2011
[3] P. R. Halmos, Measure Theory, East-West Press Pvt. Ltd., 1978
[4] M. E. Taylor, Measure Theory, AMS, 2006

MATH14-303(c): Topology - II

Quotient spaces, identification maps, cones, suspensions, local compactness and one-point compactification. proper maps, regularity, complete regularity, the Stone-Cech compactification, normality, Urysohn lemma, Tietze extension theorem, Urysohn metrization theorem, Nagata-Smirnov metrisation theorem, paracompactness, characterizations of paracompactness in regular spaces, partition of unity.

Reference books.

[1] J. Dugundji, Topology, Allyn and Bacon, 1970.
[2] R. Engelking, General Topology, Heldermann, 1989.
[3] J.L. Kelley, General Topology, Springer-verlag, 2005.
[4] J.R. Munkres, Topology, Second Edition, Pearson Education, 2003.
[5] T. B. Singh, Elements of Topology, CRC Press, 2013.
[6] S. Willard, General Topology, Dover Publications, Inc. N.Y., 2004.

Math14-304(a): Coding Theory

The communication channel, The coding problem, Types of codes, Block codes, Error-detecting and error-correcting codes, Linear codes, The Hamming metric, Description of linear block codes by matrices, Dual codes, Standard array, Syndrome.

Step-by-step decoding, Modular representation, Error-correction capabilities of linear codes, Bounds on minimum distance for block codes, Plotkin bound, Hamming sphere packing bound, Varshamov-Gilbert-Sacks bound.

Bounds for burst-error detecting and correcting codes, Important linear block codes, Hamming codes.

Golay codes, Perfect codes, Quasi-perfect codes, Reed-Muller codes, Codes derived from Hadamard matrices, Product codes, Concatenated codes.

Text book(s).

[1] W.W. Peterson and E.J. Weldon, Jr., Error-Correcting Codes. M.I.T. Press, Cambridge, Massachusetts, 1972.

Reference books.

[1] Raymond Hill, A First Course in Coding Theory, Oxford University Press, 1990.
[2] Man Young Rhee, Error Correcting Coding Theory, McGraw Hill Inc., 1989.
[3] F.J. Macwilliams and N.J. A. Sloane, The Theory of Error Correcting Codes, North- Holland, 2006.

Math14-304(b): Computational Fluid Dynamics

Mathematical description of the physical phenomena. Governing equations-mass, momentum, energy, species. General form of the scalar transport equation, Elliptic, parabolic and hyperbolic equations. Basics of discretization methods: explicit and implicit approaches. Methods for deriving discretization equations by finite differences to one-dimensional and two-dimensional parabolic, elliptic and hyperbolic equations. Schmidt, Dufort-Frankel, Lax-Wendroff, Crank-Nicolson and ADI methods.

Methods for solving discretized equations. Accuracy, stability and convergence of the finite difference methods. Methods for deriving discretization equations by finite volume methods. Convection and Diffusion- Steady one-dimensional convection and diffusion, upwind, exponential, hybrid, power, QUICK schemes.

Two-dimensional convection-diffusion, accuracy of upwind scheme; false diffusion and dispersion, boundary conditions. Flow field calculation, pressure-velocity coupling, vorticity-stream function formulation, staggered grid, SIMPLE, SIMPLER and PISO algorithms.

Finite volume methods for unsteady flows; One-dimensional unsteady heat conduction, implicit method for two-dimensional problem. Discretization of transient convection-Diffusion equation.

Text book(s).

[1] D.A. Anderson J.C. Tannehill and Richard H. Pletcher, Computational Fluid Mechanics and Heat Transfer, Taylor and Francis, Hemisphere Pub. Comp., USA, 1997.

Reference books.

[1] John D. Anderson, Computational Fluid Dynamics, McGraw-Hill, 1995.
[2] S. V. Patankar, Numerical Heat Transfer and Fluid Flow, Taylor and Francis, Hemisphere Pub. Comp., USA, 2004.
[3] H.K.Versteeg, and W.Malalasekera , An Introduction to Computational Fluid Dynamics: The Finite Volume Method, Pearson, 2007.
[4] T.J. Chung, Computational Fluid Dynamics, Cambridge Univ.Press, Newyork, USA, 2002.

Math14-304(c): Computational Methods for ODEs

Initial Value Problems (IVPs) for the system of ordinary differential equations (ODEs); Difference equations; Numerical Methods; Local truncation errors; global truncation error; Stability analysis; Interval of absolute stability; Convergence and consistency.

Single-step Methods: Taylor series method; Explicit and Implicit Runge-Kutta methods and their stability and convergence analysis; Extrapolation method; Runge-Kutta method for the second order ODEs and Stiff-system of differential equations.

Multi-step Methods: Explicit and Implicit multi-step methods; General linear multi-step methods and their stability and convergence analysis; Adams-Moulton method; Adams-Bashforth method; Nystorm method; Multi-step methods for the second order IVPs.

Boundary Value Problems(BVPs): Two point non-linear BVPs for second order ordinary differential equations; Finite difference methods; Convergence analysis; Difference scheme based on quadrature formula; Difference schemes for linear eigen value problems; Mixed boundary conditions; Finite element methods; Assemble of element equations; Variational formulation of BVPs and their solutions; Galerikin method; Ritz method; Finite element solution of BVPs;

Note: Use of scientific calculator is allowed in theory examination

Credit hours: 04 Theory + 02 practical per week.

Text book(s).

[1] J.C. Butcher, Numerical Methods for Ordinary Differential Equations, John Wiley & Sons, New York, 2003.

Reference books.

[1] J.D. Lambert, Numerical Methods for Ordinary Differential Systems: The Initial Value Problem, John Wiley and Sons, New York, 1991.
[2] K. Atkinson, W.Han and D.E. Stewart, Numerical Solution of Ordinary Differential Equations, John Wiley, New York, 2009.

MATH14-304(d): Mathematical Programming

Existence theorems, First order optimality conditions and second order optimality conditions for unconstrained optimization problems, Ekeland's variational principle

Convex functions, Differentiable convex functions, Optimization on convex sets, Separation theorems, Fritz John optimality conditions for constrained nonlinear programming problems, Constraint qualifications, Karush Kuhn Tucker conditions in nonlinear programming, Second order conditions in nonlinear programming

Lagragian saddle points, Duality in nonlinear programming, Strong duality in convex programming, duality for linear and quadratic problems.

Quadratic programming, Wolfe's method as application of Karush Kuhn Tucker conditions, convex simplex method, Penalty function methods.

Text book(s).

[1] Mokhtar S. Bazaraa, Hanif D. Sherali and C.M. Shetty, Nonlinear Programming: Theory and Algorithms, John Wiley & Sons, 2006.
[2] Osman Gler, Foundations of Optimization, Springer 2010.
[3] David G. Luenberger and Yinyu Ye, Linear and Nonlinear Programming, Springer, 2008.

Reference books.

[1] Jan Brinkhuis and Vladimir Tikhomirov, Optimization : Insights and Applications, Princeton University Press, 2005.
[2] Kenneth Lange, Optimization, Springer 2013.

MATH14-304(E): Methods of Applied Mathematics

Dimensional analysis, Buckingham Pi Theorem, Scaling, Perturbation methods, regular perturbations, singular perturbations, WKB approximations,

Integral equation: introduction and relation with linear differential equation. Volterra integral equations and its solutions: Method of resolvent kernel, Method of successive approximations. Convolution type of equation: Method of Laplace Transform, System of volterra integral equations, Integro-differential equation. Abel's integral equation and its generalizations.

Fredholm integral equations and its solutions: Method of resolvent kernels, Method of successive approximations. Integral equations with degenerate kernels, Eigen values and eigen functions and their properties, Hilbert Schmidt theorem, Non homogeneous Fredholm integral equation with symmetric kernel, Fredholm alternative.

Variational problems. the variation of a functional and its properties, Extremum of functional, Necessary condition for an extremum, Euler's equation and its generalization, the variational derivative, General variation of a functional and variable end point problem, sufficient conditions for the extremum of a functional.

Text book(s).

[1] M.L. Krasnov, Problems and exercises integral equations, Mir Publication Moscow, 1971
[2] M. Gelfand and S.V. Fomin, Calculus of variations, Prentice Hall, Inc., 2000.

[3] D. Logan: Applied mathematics: A contemporary approach, John Wiley and Sons, New York, 1997.

Reference books.

[1] F.B. Hildebrand, Methods of applied mathematics, Dover Publication, 1992.

MATH14-304(F): Graph Theory

Graphs: Vertices of graphs, Walks and connectedness, Degrees, Operations on graphs, Blocks, Cut-points, bridges and blocks, Block graphs and cut-point graphs

Trees: Elementary properties of trees, Centers and Centroids, Block-cut point trees, Independent cycles and cocycles

Connectivity and Traversability: Connectivity and line connectivity, Menger's theorems, Eulerian graph, Hamiltonian graphs

Planarity and Coloring: Planar graphs, outer planar graphs, Kuratowski's theorem, dual graphs, chromatic number, five color theorem

Text book(s).

[1] F. Harary, Graph theory, Narosa Publishing House, New Delhi, 1988.

Reference books.

[1] R. Balakrishnan and K. Renganathan, A textbook of Graph theory, Springer, 2000
[2] Bela Bollobas, Modern Graph Theory Springer, 2002
[3] G. Chartrand, L. Lesniak, Graphs & digraphs. Fourth edition. Chapman & Hall/CRC, 2005.
[4] Robin J. Wilson, Introduction to Graph Theory (4th Edition), Addison Wesley, 1996

Semester IV

MATH14-401(A): Algebraic Number Theory

Algebraic Numbers, Conjugates and Discriminants, Algebraic Integers, Integral Bases, Norms and Traces, Rings of Integers, Quadratic Fields, Cyclotomic Fields.

Trivial Factorizations, Factorization into Irreducibles, Examples of Non-Unique Factorization into Irreducibles, Prime Factorization, Euclidean Domains, Euclidean Quadratic Fields, Consequences of Unique Factorization, The Ramanujan-Nagell Theorem, Prime Factorization of Ideals.

The Norm of an Ideal, Nonunique Factorization in Cyclotomic Fields, Lattices, The Quotient Torus, Minkowski's Theorem, The Two-Squares Theorem, The-Four Squares Theorem, The Space Lst

The Class-Group, An Existence Theorem, Finiteness of the Class-group, How to Make an Ideal Principal, Unique Factorization of Elements in an Extension Ring, Factorization of a Rational Prime, Minkowski's Constants, Some Class-Number Calculations.

Text book(s).

[1] I. N. Stewart and D. O. Tall, Algebraic Number Theory, Chapman and Hall, London, 1987.

Reference books.

[1] K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, Springer-Verlag, 1990.
[2] S. Lang, Algebraic Number Theory, Springer-Verlag, New York Inc., 1994.
[3] D. A. Marcus, Number Fields, Springer-Verlag, New York Inc., 1987.

MATH14-401(b): Theory of Non-Commutative Rings

Basic terminology and examples: simple rings, Dedekind-finite rings, opposite rings, rational quaternions, rings with generators and relations: polynomial rings in commuting variables, group and semi group ring, formal power series ring with pairwise commuting variables or otherwise, division ring of formal Laurent series, hilbert's twist, differential polynomial rings, triangular rings, example of one-sided Noetherian, Artinian rings, semisimple rings, structure of semisimple rings, Wedderburn-Artin's theorem, structure theorem of simple Artinian rings, Constructions of Non-Artin simple rings using skew polynomial and skew Laurentz polynomial, Jacobson radical, prime radical, Jacobson semi simple rings, Hopkins Levitzki theorem, Von Newmann regular rings, prime and semiprime rings, structure of primitive rings, density theorem, direct products, subdirect sums, commutativity theorems, local rings and semi-local rings.

Text book(s).

[1] T. Y. Lam, A first course in Non-Commutative Rings, Springer-Verlag, 1991.

Reference books.

[1] I. N. Herstein, A First Course in Non-commutative Rings, Carus Monographs of AMS, 1968.
[2] Louis H. Rowen, Ring Theory, Academic Press, 1991.
[3] T. W. Hungerford, Algebra, Springer-Verlag, New York, 1981.

MATH14-401(c): Simplicial Homology Theory

Geometric simplexes, geometric complexes and polyhedra. Simplicial maps, barycentric subdivision, simplicial approximation of continuous maps, contiguous maps. Orientation of geometric complexes, homology groups. Computation of homology groups, the homology of n-sphere. The structure of homology groups, the chain complexes, chain mappings, chain derivation, chain homotopy. The homomorphism induced by continuous maps between two polyhedra. Functorial property of induced homomorphisms, Topological invariance of homology groupsThe degree of self mappings of S^n. The Brower's fixed point theorem, Brower's degree Theorem, Euler-Poincare theorem, Euler's formula, Lefschetz fixed point theorem. Existence of eigen value, Relative homology groups. Invariance of dimension.

Reference books.

[1] H Agoston, Algebraic Topology, Marcel Dekker, 1976.
[2] M A Armstrong,Basic Topology, Springer-Verlag, 1983.
[3] F H Croom,Basic concepts of Algebraic Topology,1976.
[4] S. Deo, Algebraic Topology, A primer, Hindustan Book Agency (2006).

MATH14-402(a): Abstract Harmonic Analysis

Introduction to representation theory of involutive Banach algebra, unitary representation of locally compact groups, Gelfand-Raikov theorem.

Representation of some special groups $SU(2)$, Lorentz group, the group of linear transformations of $\mathbb{R}$, unitary representation of compact groups, Schur's lemma, the orthogonality relations.

Characters of finite dimensional representations, Weyl-Peter theorem, convolution of bounded regular complex measures.

The convolution of Banach algebra $M(G)$, Fourier-Stieltjes transform, positive definite functions, Bochner's theorem.

Text book(s).

[1] E. Hewitt and K.A. Ross, Abstract Harmonic Analysis, Vol I, Springer-Verlag, 1993.

Reference books.

[1] J. M. G. Fell and R. S. Doran, Representation of * algebras, Locally Compact groups and Banach * Algebraic Bundles Vol I, II, Academic Press, 1988.
[2] G. B. Folland, A Course in Abstract Harmonic Analysis, CRC Press, 1995.
[3] W. Rudin, Fourier Analysis on Groups, Interscience Publisher, 1990.

MATH14-402(b): Frames and Wavelets

Finite frames, canonical reconstruction formula, frames and matrices, similarity and unitary equivalence of frames, frame bounds and frame algorithms.

Frames and Bessel sequences in infinite dimensional Hilbert spaces, frame sequence, the Gram matrix. Frames and operators, characterization of frames, dual frames, tight frames, continuous frames, frames and signal processing.

Riesz bases, Frames versus Riesz bases, conditions for a frame being a Riesz basis, frames containing a Riesz basis. Bases in Banach spaces, Limitations of bases.

Wavelets, Haar wavelets, basic properties of the Haar scaling function, Haar decomposition and reconstruction algorithms. The Daubechies wavelets, wavelet bases, scaling function.

Text book(s).

[1] O. Christensen, An introduction to frames and Riesz bases, Birkhaüser (2003)
[2] S. Mallat, A wavelet tour of signal processing, Academic Press (2009).

Reference books.

[1] D. Han, K. Kornelson, D. Larson and E. Weber, Frames for undergraduates, Student Math. Lib., (AMS) Vol. 40 (2007).
[2] A. Boggess and F. J. Narcowich, A first course in wavelets with Fourier analysis, Wiley (2009).
[3] D. F. Walnut, Wavelet analysis, Birkhaüser (2002)

MATH14-402(c): Operators on the Hardy-Hilbert Spaces

The Hardy Hilbert Space: Basic Definitions and properties.

The unilateral shift and factorisation of Spectral structure. functions: Shift operators, Invariant and reducing subspaces. Inner and outer factorisation, Blaschke factors, singular inner functions, outer functions.

Toeplitz operators: Basic properties of Toeplitz operators,spectral structure.

Hankel operators: Bounded Hankel operators, Hankel operators of finite rank, Compact Hankel operators, self adjointness and normality of Hankel operators. Relation between Hankel and Toeplitz operators.

Text book(s).

[1] R.A. Martinez-Avedano and P. Rosenthal, An Introdution to the Hardy Hilbert Space, Graduate Texts in Mathematics 237, Springer, 2007.

Reference books.

[1] R.G. Douglas, Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics 179, Springer, 1998
[2] N.K. Nikolskii, Operators, Functions and Systems: An Easy Reading, Volume 1, Mathematical Surveys and Monographs 92, American Mathematical Society, 2002.

MATH14-402(d): Theory of Unbounded Operators

Unbounded linear operators and their Hilbert Adjoints, Hellinger-Toeplitz Theorem, Hermitian, symmetric and self-adjoint linear operators.

Closed linear operators, closable operators and their closures on Banach spaces, Cayley transform, definciency indices.

Spectral properties of self-adjoint operators; Multiplication and differentiation operators and their spectra.

Semigroup of bounded linear operators: Uniformly continuous and Strongly continuous semigroups, generator of a semigroup, Hille-Yosida Theorem. Dissipative operators, Lumer-Phillip Theorem, properties of dissipative operators. Group of bounded linear operators, Stones Theorem.

Text book(s).

[1] E. Kreyszig, Introductory Functional Analysis with Applications,John Wiley and Sons (2001) and A. PAzy, Semigroups of Linear Operators and Applications to Partial Differential Equations, Springer (1983)

Reference books.

[1] S. Goldberg, Unbounded Linear Operators : Theory and Applications, Dover, (2006)

[2] M. Schechter, Principles of Functional Analysis, American Mathematical Society, (2004)

MATH14-403(a): Calculus on $\mathbb{R}^n$

The differentiability of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, partial derivatives and differentiability, directional derivatives and differentiability, chain rule, mean value theorems, inverse function theorem and implicit function theorem.

Derivatives of higher order, Taylor's formulas with integral reminder, Lagrange's reminder and Peano's reminder; Integration over a k-cell, primitive mappings, partition of unity, change of variables.

Introduction to differential forms on $\mathbb{R}^n$, basic properties of differential forms, differentiation of differential forms, change of variables in differential forms.

Simplexes and chains, integration of differential forms, Stokes' theorem.

Text book(s).

[1] J.R. Munkres, Analysis on Manifolds, Addison Wesley, 1997
[2] W. Rudin, Principles of Mathematical Analysis, 3rd Edition, Mc Graw Hill, 1986

Reference books.

[1] M. Giaquinta and G. Modica, Mathematical Analysis, An introduction to functions of several variables, Birkhauser, 2009
[2] M. Spivak, Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus, Westview Press, 1998.

MATH14-403(B): DIFFERENTIAL GEOMETRY

Graph and level sets, vector fields, the tangent space, surfaces, orientation, the Gauss map, geodesics, parallel transport, the Weingarten map, curvature of plane curves, arc length and line integrals, curvature of surfaces, parametrized surfaces, surface area and volume, surfaces with boundary, the Gauss-Bonnet Theorem.

Reference books.

[1] Wolfgang Kuhnel: Differential Geometry - curves-surfaces- Manifolds. Second Edition, 2006, AMS.
[2] A: Mishchenko and A. Formentko. A course of Differential Geometry and topology) Mir Publishers Moscow, 1988.
[3] Andrew Pressley: Elementary Differential Geometry. SUMS (Springer), 2001 (Ist Indian Repring 2004).
[4] I.A. Thorpe: Elementary Topics in Differential Geometry. Springer, 1979 (Ist Indian Reprint 2004).

MATH403(C): TOPOLOGICAL DYNAMICS

Dynamical Systems: Definition and examples (including real life examples), Orbits, Types of orbits, Topological conjugancy and orbits, Phase Portrait - Graphical Analysis of orbit, Periodic points and stable sets, Omega and alpha limit sets and their properties, Sarkoviskii's Theorem, Dynamics of Logistic Functions, Shift spaces and subshifts, Subshift of finite type and subshift represented by a matrix.

Definition and examples of expansive homeomorphisms, Properties of expansive homeomorphisms, Non-existence of expansive homeomorphism on the unit interval and unit circle, Generators and weak generators, Generators and expansive homeomorphisms, Converging semiorbits for expansive homeomorphisms, Definition and examples of shadowing property, properties of shadowing property, Topological Stability, Anosov maps and topological stability.

Practicals using MATLAB

Reference books.

[1] N. Aoki and K. Hiraide, Topological theory of Dynamical Systems, Recent Advances, North Holland Publications, 1994.

[2] Brin and Stuck, Introduction to Dynamical Systems,, Cambridge Univ. Press, 2002

[3] D. Hanselman and B. Littlefiels, Mastering MATLAB, Pearson Education, 2005.

[4] D. Lind and B. Marcus, Symbolic Dynamics and Coding, Cambridge University Press, 1996.

[5] Clark Robinson, Dynamical Systems, Stability, Symbolic Dynamics and Chaos, CRC Press, 1998.

[6] J. De. Vries, Elements of Topological Dynamics, Mathematics and its applications, Kluwer Academic Publishers, 2000

MATH14-404(A): ADVANCED CODING THEORY

Tree codes, Convolutional codes, Description of linear tree and convolutional codes by matrices, Standard array, Bounds on minimum distance for convolutional codes, V-G-S bound, Bounds for burst-error detecting and correcting convolutional codes, The Lee metric, Packing bound for Hamming code w.r.t. Lee metric, The algebra of polynomials, Residue classes, Galois fields, Multiplicative group of a Galois field.

Cyclic codes, Cyclic codes as ideals, Matrix description of cyclic codes, Hamming and Golay codes as cyclic codes, Error detection with cyclic codes, Error-correction procedure for short cyclic codes, Shortended cyclic codes, Pseudo cyclic codes.

Code symmetry, Invariance of codes under transitive group of permutations, Bose-Chaudhary-Hocquenghem (BCH) codes, BCH bounds, Reed-Solomon (RS) codes, Majority-logic decodable codes, Majority-logic decoding.

Singleton bound, The Griesmer bound, Maximum-distance separable (MDS) codes, Generator and parity-check matrices of MDS codes, Weight distribution of MDS code, Necessary and sufficient conditions for a linear code to be an MDS code, MDS codes from RS codes, Abramson codes, Closed-loop burst-error correcting codes (Fire codes), Error locating codes.

Text book(s).

[1] F.J. Macwilliams and N.J. A. Sloane, Theory of Error Correcting Codes, North- Holland Publishing Company, 2006.

[2] W.W. Peterson and E.J. Weldon, Jr., Error-Correcting Codes, M.I.T. Press, Cambridge, Massachusetts, 1972.

Reference books.

[1] E.R. Berlekamp, Algebraic Coding Theory, McGraw Hill Inc., 1984.

[2] W.C. Huffman and V. Pless, The Theory of Error Correcting Codes, CambridgebUniversity Press, 1998.

MATH14-404(b): Advanced Fluid Mechanics

Thermodynamics and dimensional analysis: Equation of state of a substance, First law of Thermodynamics, Internal energy and specific heat of gas, entropy, Second law of thermodynamics. Energy equation, Non-dimensionalizing the basic equations of incompressible viscous fluid flow, Non-dimensional Numbers.

Gas Dynamics: Compressibility effects in real fluids, Elements of wave motion in a gas, Speed of sound, Basic equation of one-dimensional compressible flow, Subsonic, sonic and supersonic flows, Isentropic gas Flow, Flow through a nozzle, Normal shock wave, oblique shock wave and their elementary analysis.

Magnetohydrodynamics: Concept, Maxwell' electromagnetic field equations, Equation of motion of a conducting fluid, MHD approximations, Rate of flow of charge, Magnetic Reynolds number and Magnetic field equation, Alfven's theorem, Magnetic body force, Magnetohydrodynamic Waves.

Boundary layer theory: Concept of Boundary Layer, Boundary layer thickness, Boundary layer equations for two-dimensional incompressible flow, Boundary layer along a flat plate, General properties of the boundary-layer equations: dependence on Reynolds number, similar solutions, Momentum and energy integral equations for the boundary layer.

Text book(s).

[1] Alan Jeffery, Magnetohydrodynamics, Oliver and Boyd Ltd., Edinburgh, 1966
[2] H. Schlichting, K. Gersten, Boundary Layer Theory, Springer, 2000
[3] F. Chorlton, Text Book of Fluid Dynamics, GK Publisher, 2009

Reference books.

[1] P.K.Kundu, I.M.Cohen,Fluid Mechanics, 5th edition, Elsevier Inc., 2012
[2] G.K.Batchelor, Introduction to Fluid Mecanics, Foundation book, New Delhi. 1994
[3] R.W.Fox, P.J.Pritchard, A.T.Mcdonald, Introduction to Fluid Mechanics, John Wiley and Sons, 2010

Math14-404(c): Computational Methods for PDEs

Finite difference methods for 2D and 3D elliptic boundary value problems (BVPs) of second and fourth order approximations; Finite difference approximations to Poissons equation in cylindrical and spherical polar coordinates; Solution of large system of algebraic equations corresponding to discrete problems and iterative methods (Jacobi, Gauss-Seidel and SOR); Alternating direction methods.

Different 2- and 3-level explicit and implicit finite difference approximations to heat conduction equation with Dirichlet and Neumann boundary conditions; Stability analysis, compatibility, consistency and convergence of the difference methods; ADI methods for 2- & 3-D parabolic equations; Finite difference approximations to heat equation in polar coordinates.

Methods of characteristics for evolution problem of hyperbolic type; Von-Neumann method for stability analysis; Operator splitting methods for 2D and 3D wave equations; Explicit and implicit difference schemes for first order hyperbolic equations and their stability analysis; System of equations for first order hyperbolic equations; Conservative form.

Finite element methods for second order elliptic BVPs; Finite element equations; Variational problems; Triangular and rectangular finite elements; Standard examples of finite elements; Mixed finite element methods; Finite element method for parabolic initial and boundary value problems; Semi-discrete and complete discrete schemes; Error estimates and convergence analysis.

Note: Use of scientific calculator is allowed in theory examination

Credit hours: 04 Theory + 02 practical per week.

Text book(s).

[1] J.C. Strickwerda, Finite Difference Schemes & Partial Differential Equations, SIAM publications, 2004.

[2] C. Johnson, Numerical solution of partial differential equations by Finite element methods, Cambridge University Press, 1987.

Reference books.

[1] A. J. Davies, The finite element method: An introduction with partial differential equations, Oxford University Press, 2011.

[2] K. W. Morton, & David Mayers, Numerical solution of partial differential equations, Cambridge University Press, 2005.

[3] J.W.Thomas, Numerical Partial Differential Equations: Finite Difference Methods, Springer and Verlag, Berlin, 1998.

[4] J.W.Thomas, Numerical Partial Differential Equations: Conservation Laws and Elliptic Equations, Springer and Verlag, Berlin, 1999.

Math14-404(d): Cryptography

Secure communications, Shift ciphers, Affine ciphers, Vigenere cipher key, Symmetric key, Public key, Block ciphers, One-time pads, Secure random bit generator, Linear feedback shift register sequences.

Differential cryptanalysis, Modes of DES, Attack on DES, Advanced encryption standard.

RSA, Attacks on RSA, Diffie-Hellman key exchange, ElGamal public key cryptosystem, cryptographic hash function.

RSA signatures, ElGamal signature, Hashing and signing, Digital signature algorithm.

Text book(s).

[1] Johannes A. Buchmann, Introduction to Cryptography, Springer 2000.

Reference books.

[1] Douglas Robert Stinson, Cryptography - Theory and Practice, Chapman Hall / CRC 2006.
[2] Wade Trappe and Lawrence C. Washington, Introduction to Cryptography with Coding Theory, Pearson Prentice Hall, 2006.

MATH14-404(E): DYNAMICAL SYSTEMS

Review of stability for linear systems. Flow defined by nonlinear systems of ODEs, linearization and stable manifold theorem. Hartman-Grobman theorem. Stability and Lyapunov functions. Saddle point, nodes, foci, centers and nonhyperbolic critical points. Gradient and Hamiltonian systems.

Limit sets and attractors. Poincare map, Poincare Benedixson theory and Poincare index Theorem. Structural stability and Peixoto's theorem, Bifurcation at non-hyperbolic equilibrium points.

Reference books.

[1] V.I. Arnold, Ordinary Differential Equations, rentice Hall of India, New Delhi, 1998.
[2] M.W. Hirsch and S. Smale, Differential Equations, Dynamical Systems and inear Algebra, Academic Press, NY, 174.
[3] L. Perko, Differential Equations and Dynamical Systems, Springer Verlag, NY, 1991.
[4] S. Wiggins, Introduction to Applied Nonlinear Dynamical Systems and Chaos, TAM Vol.2, Springer-Verlag, NY, 1990.
[5] Richard Holmgren, A First Course in Discrete Dynamical Systems, Springer-Verlag, NY, 1994.

MATH14-404(F): OPTIMIZATION TECHNIQUE AND CONTROL THEORY

Extended real valued functions, Proper convex functions, Subgradients, Directional derivatives.

Conjugate functions, Dual convex programs, Optimality conditions and Lagrange multipliers, Duality and optimality for standard convex programs, Gradient descent method, Gradient projection method.

Newton's method, Conjugate gradient method, Dynamic programming, Bellman's principle of optimality, Allocation problem, Stage coach problem.

Optimal control problem and formulations, Variational approach to the fixed-time free endpoint problem, Pontryagin's maximum principle, Dynamic programming and Hamilton-Jacobi-Bellman equation.

Reference books.

[1] Mordecai Avriel, Nonlinear Programming: Analysis & Methods, Dover Publications, New York, 2003.

[2] Osman Gler, Foundations of Optimization, Springer 2010.

[3] Frederick S. Hillier and Gerald J. Lieberman, Introduction to Operations Research, McGraw-Hill, 2010.

[4] Daniel Liberzon, Calculus of Variations and Optimal Control Theory: A Concise Introduction, Princeton University Press, 2012.

[5] Jan Brinkhuis and Vladimir Tikhomirov, Optimization : Insights and Applications, Princeton University Press, 2005.

[6] Kenneth Lange, Optimization, Springer 2013.

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF DELHI
DELHI – 110007

Tel. No. 27666658

M.Phil (Full-time) Programme in Mathematics
(Effective from 2014-2015 onwards)

ELIGIBILITY. The candidate should have good academic record with first or high second class Master's Degree or an equivalent degree of a foreign University in the subject concerned, or an allied subject to be approved by the Vice-Chancellor on the recommendation of the Head of the Department and the Dean of the Faculty concerned. Relaxation to the reserved category students will be provided as per University rules.

ADMISSION PROCEDURE. Admission to the M.Phil programme will be done on the basis of the relative merit of student's performance at post-graduate examination and an entrance test to be conducted by the Department. The merit list will be prepared by taking into account 25% of marks scored in post-graduate examination and 75% of marks scored in the entrance test.

ALLOCATION OF SUPERVISOR. The allocation of the supervisor for a selected student shall be decided by the M.Phil Committee in a formal manner depending on the number of student per faculty member, the available specialization among the faculty supervisors, and the research interest of the student as indicated during counseling by the student. The allotment/allocation of supervisor shall not be left to the individual student or teacher. The M.Phil committee shall also constitute an Advisory Committee of 3 members including the Supervisor of the candidate.

PROGRAMME DESCRIPTION. The M.Phil Programme shall consist of two parts: Part I: Course work and Part II: Dissertation.

Part-I: Course Work. A student shall be required to study four courses selecting at least one from different group listed below. However, the advisory committee may prescribe a specialized course not listed below in lieu of one of the four. Such a course will be offered by the supervisor(s) after providing the details of the course to the M.Phil. committee; the examination will be held in the same manner as of the other papers.

Group- A

- i) Distribution Theory and Calculus on Banach Spaces
- ii) Operator Theory and Function Spaces
- iii) Geometric Function Theory
- iv) Introduction to Operator Algebras
- v) Advanced Frame Theory

Group- B

- i) Rings and Modules
- ii) Group Rings
- iii) Advanced Commutative Algebra
- iv) Differential Manifolds

v) Topological Dynamics
vi) Topological Structures
vii) Chaos Theory
viii) Ergodic Theory
ix) Singular Homology Theory

Group- C

i) Graph and Network Theory
ii) Convex and Nonsmooth Analysis / Multi-objective Optimization
iii) Combinatorial Mathematics
iv) Parallel Iterative methods for Partial Differential Equations
v) Hydrodynamic Stability Theory
vi) Advanced Compressible Flows
vii) Elliptic Curves and Cryptography

The candidates will be examined in their coursework out of 300 marks and each course will carry 75 marks. Each course will be of one semester duration with 3 lectures per week.

Part II: Dissertation. After passing the examination in the courses as above, a student shall be required to write a dissertation on a subject approved by the Advisory Committee under the supervision of the Supervisor(s) appointed for the purpose. A student who has secured 50% marks in at least two courses of the Part I Examination may be allowed to proceed for his dissertation work. Such a student shall be permitted to submit his dissertation only when he/she has passed the examination in all the courses prescribed under Part I.

The Dissertation may include results of original research, a fresh interpretation of existing facts and data or a review article of a critical nature or may take such other form as may be determined by the Advisory Committee. The dissertation will be submitted only when the Supervisor(s) concerned is/are satisfied that the Dissertation is worthy of consideration in partial fulfillment of the M.Phil. Degree.

Title of the dissertation should be approved and the appointment of external examiner be made before the submission of the dissertation. The application for approval of the title must include a synopsis together with a list of selected main references.

DURATION. The duration of the Programme will be one and half years and the total span period is 36 months. There shall be no provision for extension of the programme duration on a case to case basis.

The dissertation can only be submitted after one year from the date of admission to the M. Phil programme subject to qualifying Part–I examination by a candidate.

No student shall be allowed to take up any assignment outside the University Department during the programme or before submission of his/her dissertation whichever is earlier.

ATTENDANCE. The minimum percentage of lectures to be attended and seminars to be participated in by the students shall be 2/3 of the lectures delivered and seminars held separately.

SCHEME OF EXAMINATION

(a) The Evaluation in each course will be based on the student's performance in written end-semester examination and internal assessment. The written examination of three hours duration will comprise of 50 marks. The internal assessment on the basis of assignment, attendance, class-room performance and seminars will be of 25 marks.

(b) Supplementary examination will be conducted for those who fail in the Part-I examination and it will be conducted within three months of the declaration of the result of Part-I examination. Students can appear in the supplementary examination only in the papers in which they fail or fail to appear. No student shall be allowed to avail more than two chances in any course of the Part-I examination.

(c) No student shall be allowed to reappear in any course of Part-I examination just to improve upon the score.

(d) The dissertation shall be evaluated by the supervisor(s) and one external examiner to be appointed by the M.Phil Committee.

(e) The total marks for Part – II examination is 200. The weightage of written dissertation will be 150 marks. Both the examiners will be required to submit marks out of 75 separately to the Head of the Department before fixing the date for viva-voce. The remaining 50 marks for the viva-voce shall be awarded jointly by both the examiners.

(f) The candidate shall submit the final dissertation after making corrections/alterations (if any) suggested by the examiners within one month of the viva-voce.

RESULT. The minimum marks required to pass the M.Phil. examination shall be 50% marks in each course of Part-I examination and 50% of marks in Part II separately. The successful candidates shall be classified as follows:

I Division with Distinction:	At least 75% marks in the aggregate.
I Division:	At least 60% marks in the aggregate but below 75% marks
Pass:	All others.

SYLLABUS

Group- A

Distribution Theory and Calculus on Banach Spaces. Test functions and distributions, some operations with distributions, local properties of distributions, convolutions of distributions, tempered distributions and Fourier transform, fundamental solutions.

The Frechet derivative, chain rule and mean value theorems, implicit function theorem, extremum problems and Lagrange multipliers.

References.

[1] W. Cheney : Analysis for Applied Mathematics; Springer -Verlag, 2001.
[2] S. Kesavan : Topics in Functional Analysis and Applications; New Age International Publishers, 2008
[3] W. Rudin : Functional Analysis; Tata Mc-Graw Hill, 1991.
[4] Robert S. Strichartz : A guide to distribution theory and Fourier transforms; World Scientific Publishing Co., 2003.

Operator Theory and Function Spaces . Fredholm operators; semi-Fredholm operators; index of a Fredholm (semi- Fredholm) Operator; essential spectrum; Weyl spectrum and Weyl theorem; direct sums of operators, their spectra and numerical ranges; weighted shifts, their norms and spectral radii; normaloid, convexoid and spectraloid operators.

Invariant subspace problem; transitive, reductive and reflexive algebras; von-Neumann algebras.

Hardy spaces: Poisson's kernel; Fatou's theorem; zero sets of functions; multiplication, composition, Toeplitz and Hankel operators.

References.

[1] Vladimir V.Peller, Hankel operators and their applications, Springer, 2002.
[2] Nikolai L.Vasilevski, Commutative algebras of Toeplitz operators on Bergman space, Birkhauser, 2008.
[3] N.Young, An introduction to Hilbert space, Cambridge University Press, 1988.
[4] P.R.Halmos, A Hilbert space problem book, II Ed., D.Van Nostrand Company, 1982.
[5] H.Radjavi and P.Rosenthal, Invariant subspaces, Springer Verlag, 1973.

Geometric Function Theory. Area theorem, growth, distortion theorems, coefficient estimates for univalent functions special classes of univalent functions. Lowner's theory and its applications; outline of de Banges proof of Bieberbach conjecture. Generalization of the area theorem, Grunsky inequalities, exponentiation of the Grunsky inequalities, Logarithmic coefficients. Subordination and Sharpened form of Schwarz Lemma

References.

[1] P. Duren, Univalent Functions, Springer, New York, 1983
[2] A. W. Goodman, Univalent Functions I & II, Mariner, Florida, 1983
[3] Ch. Pommerenke, Univalent Functions, Van den Hoek and Ruprecht, Göttingen, 1975.
[4] M. Rosenblum, J. Rovnyak, Topics in Hardy Classes and Univalent Functions, Birkhauser Verlag, 1994
[5] D. J. Hallenbeck, T. H. MacGregor, Linear Problems and Convexity Techniques in Geometric Function Theory, Pitman Adv. Publ. Program, Boston-London-Melbourne,1984.
[6] I.Graham, G. Kohr, Geometric Function Theory in One and Higher Dimensions, Marcel Dekker, New York, 2003.

Introduction to Operator Algebras. Basic definitions and examples of Banach*-algebras, Spectrum of a Banach algebra element, L^1-algebras and Beurling algebras, Tensor products of Banach algebras, Multiplicative linear functional, The Gelfand representations, Fourier algebra, Functional calculus of in C*-algebras, Continuity and homomorphisms, Approximate identities in C*-algebras, Quotient algebras of C*-algebras, Representations and positive linear functional, Double Commutation Theorem, Enveloping von Neumann algebra of a C*-algebra, Tensor products of C*-algebras.

References.

[1] J.Diximier, C*-algebras, North-Holland Amersdem, 1977.
[2] R.V. Kadison and J.R.Ringrose, Fundamentals of the theory of operator algebras, Graduate studies in Mathematics, 15, AMS, Providence, 1997.
[3] E.Kaniuth, A course in commutative Banach algebras, Springer Verlag, 2008.
[4] M.Takesaki, Theory of Operator algebras, Springer Verlag, 2001.

Advanced Frame Theory. An overview on frames. B-Spline Symmetric B-Splines. Frames of translates. The canonical dual frame . Compactly supported generators. An application to sam-pling theory. Shift-Invariant Systems, Frame-properties of shift-invariant system, Representations of the frame operator. Gabor Frames in $L^2(R)$. Basic Gabor frame theory, Tight Gabor frames, The duals of a Gabor frame, Explicit construction of dual frame pairs, Popular Gabor conditions Representations of the Gabor frame operator and duality. Wavelet frames in $L^2(R)$.

References.

[1] O. Christensen, Frames and bases (An introductory course), Birkhauser, Boston (2008).
[2] I. Daubechies, Ten Lectures on wavelets, SIAM, Philadelphia (1992).
[3] R. Young, A introduction to non-harmonic Fourier series, Academic Press, New York (revised edition 2001).

Group- B

Rings and Modules.Essential and superfluous submodules, Decomposition of rings, Generating and cogenerating, Modules with composition series, Fitting Lemma, Indecomposable decompositions of modules, Projective modules and generators, Radicals of projective modules, Projective covers, Injective hulls, Cogenerators, Flat modules. Singular submodules, Localization and maximal quotient rings. Essential finite generation, Finite dimensionality, Uniform modules and Goldie rings. Regular rings, Strongly regular rings, Unit regular rings, Right π- regular rings. Baer rings, Rickart rings. Baer*rings, Rickart*rings.

References.

[1] A.F.Anderson and K.R.Fuller: Rings and categories of modules, Springer- Verlag,1991 (Relevant sections of Ch. 2,3,4,5).
[2] S.K.Berberian : Baer Vings,Springer Verlag, New York ,1972 (Ch.1, sections 3, 4).
[3] K.R.Goodeari : Ring theory (Non singular rings and modules), Marcel Dekker,Inc. New York (Relevant sections of Ch. 1,2,3).
[4] K.R.Goodeari : Von Neumann regular rings,Pitman, London, 1979 (Ch. 1,3,4).
[5] T.Y.Lam: Lectures on Modules and rings, Springer Verlag, 1998(Ch. 3 ,section 7(d)).

Group Rings. Twisted Group Rings, Tensor Products, Idempotents, Finite groups, Augmentation annihilators, Group algebra as injective modules, Linear identities. The Center, Finite conjugate groups, Chain conditions.

References.

[1] D. S. Passman The Algebraic structure of Group Rings, Dover Publications (Reprint edition). 2011.
[2] S. K. Sehgal, Topics in Group Rings, Marcel Dekker, New York, and Basel, 1978.
[3] I.B.S. Passi, Group Rings and their Augmentation Ideals Lecture Notes in Mathematics 715, Springer, New York, 1979.
[4] A. A. BOVDI, Group Rings Uzhgorod State University, 1978.
[5] D. S. Passman, Infinite Group Rings, Pure and Applied Math. 6, Marcel Dekkar, New York, 1971.
[6] P. Rihenboim, Rings and Modules, Interscience Tracts in Pure and Applied Mathematics, No.6, Interscience, New York, 1969.
[7] C.P. Milies and S.K. Sehgal, An Introduction to Group Rings, Kluwer Academic Publishers, Dordrecht, 2002.

Advanced Commutative Algebra. Direct limit, Inverse limit, Graded rings and modules, Associated graded rings, I-adic completion, Krull's intersection theorem, Hensel's lemma, Hilbert function, Hilbert polynomial, Dimension theory of Noetherian local rings, Regular local rings, UFD property of regular local rings, Hom functor, Tensor functor, I-torsion functor, Flat modules, Projective and injective modules, Complexes, Projective and injective resolution, Derived functor, Tor and ext functor, Minimal resolution, Regular sequences, Cohen-Macaulay rings and modules.

References:

[1] H. Matsumura, Commutative ring theory, Cambridge university press, 1989.
[2] Balwant Singh, Basic commutative algebra, World scientific publishing co., 2011.
[3] D. Eisenbud, Commutative algebra with a view towards algebraic geometry, Springer verlag, 1995.
[4] M.F. Atiyah & I.G. Macdonald, Introduction to commutative algebra, Addison Wesley, 1969.

Differential Manifolds. The derivative, continuously differentiable functions, the inverse function theorem, the implicit function theorem. Topological manifolds, partitions of unity, imbeddings and immersions, manifolds with boundary, submanifolds. Tangent vectors and differentials, Sard's theorem and regular values, vector fields and flows, tangent bundles, smooth maps and their differentials. Smooth manifolds, smooth manifolds with boundary, smooth sub-manifolds, construction of smooth functions.

References.

[1] G.E. Bredon, Topology and Geometry, Springer-verlag, 1993.
[2] L. Conlon, Differentiable Manifolds, Second Edition, Birkhauser,2003.
[3] A. Kosinski, Differential Manifolds, Academic Press, 1992.
[4] M. Spivak, A Comprehensive Introduction to Differential Geometry, Vol. I; Publish or Perish, 1979.

Topological Dynamics. Dynamical Systems: Definition and examples (including real life examples), Orbits, Types of orbits, Topological conjugancy and orbits, Phase Portrait - Graphical Analysis of orbit, Periodic points and stable sets, Omega and alpha limit sets and their properties, Sarkoviskii's Theorem, Dynamics of Logistic Functions, Shift spaces and subshifts, Subshift of finite type and subshift represented by a matrix.

Definition and examples of expansive homeomorphisms, Properties of expansive homeomorphisms, Non-existence of expansive homeomorphism on the unit interval and unit circle, Generators and weak generators, Generators and expansive homeomorphisms, Converging semiorbits for expansive homeomorphisms, Definition and examples of shadowing property, properties of shadowing property, Topological Stability, Anosov maps and topological stability.

REFERENCE BOOKS:

[1] N. Aoki and K. Hiraide, Topological theory of Dynamical Systems, Recent Advances, North Holland Publications, 1994.
[2] D. Lind and B. Marcus, Symbolic Dynamics and Coding, Cambridge University Press, 1996.
[3] Clark Robinson, Dynamical Systems, Stability, Symbolic Dynamics and Chaos, CRC Press,1998.
[4] J. De. Vries, Elements of Topological Dynamics, Mathematics and its applications, Kluwer Academic Publishers, 2000.
[5] D. Hanselman and B. Littlefiels, Mastering MATLAB, Pearson Education, 2005.
Introduction to Dynamical Systems, Brin and Stuck, Cambridge Univ. Press, 2002

Topological Structures. Dimension Theory: Definition and basic properties of the three dimension function inc. Inc. and dim, Characterization and subset theorems, equality of dim X and dim βX equality of Ind X and Ind βX.

Paracompactness: Paracompactness and full normality, presentation of paracompactness under mappings, Hanai-Moritastone theorem, products of paracompact spaces, countable paracompactness, strong paracompactness characterizations of strong paracompactness in regular spaces, products and subspaces of strongly paracompact spaces, pointwise paracompactness Arens Dugundji theorem, collectionwise normal spaces, Ding's example of a normal space which is not collectionwise normal.

Bitopological Spaces: Basic concepts, subspaces and products Separation and covering axioms.

References.

[1] R. Engelking: General Topology, Polish Scientific Publishers Warszawa, 2nd Ed., 1977.
[2] K. Nagami: DimensionTheory, Academic Press, New York, 1970.
[3] W.J. Pervin: Foundations of General Topology, Academic Press Inc., New York, 1964.
[4] S. Willard: General Topology, Addison-Wesley Publishing Co. Inc., 1970.

Chaos Theory. Topological transitivity: Examples and properties, Topological mixing: Examples and Properties, Transitivity and limit sets for maps on I , Characterizing topological mixing in terms of topological transitivity for maps on I , Sensitive dependence on initial conditions, Devaney's definition of chaos, Logistic maps and shift maps as chaotic maps, Period three implies chaos, Relation between transitivity and chaos on I .

Topological Entropy: Definitions, Entropy of interval maps, Horseshoes, Entropy of cycles, Continuity properties of the Entropy, Entropy of shift spaces, Entropy for circle maps, Various other definitions of Chaos and their interrelationships.

References.

[1] L. Alseda, J. Llibre, M. Misiurewicz, Combinatorial Dynamics and Entropy inDimension One, Advanced Series in Nonlinear Dynamics, 2000.
[2] L. S. Block and W. A. Coppel, Dynamics in One dimension, Springer, 1992.
[3] R. L. Devaney, A First Course in Chaotic Dynamical Systems, Westview Press, 1992.
[4] D. Hanselman and B. Littlefiels, Mastering MATLAB, Pearson Education,2005.
[5] Clark Robinson, Dynamical Systems, Stability, Symbolic Dynamics and Chaos, CRC press, 1999.
[6] S. Ruette, Chaos for continuous interval maps: A survey of relationship between various sorts of chaos, 2003.
[7] Introduction to Dynamical Systems, Brin and Stuck, Cambridge Univ. Press, 2002

Ergodic Theory. Measure preserving transformations and examples, Recurrence, Poincare's Recurrence theorem, Ergodicity, ergodicity of shift transformations

Ergodic theorems of Birkhoff and Von Neuman, Mixing, Weak-mixing and their characterizations, the isomorphism problem: conjugacy, Spectral equivalence, Transformations with discrete spectrum, Entropy, Kolmogorov-Sinai theorem, K-systems examples of calculation of entropy, Unique ergodicity, uniformly distributed sequences, applications to Diophantine approximation.

References.

[1] P. R. Halmos, Lectures on Ergodic Theory, American Mathematical Society, 2006
[2] M. G. Nadkarni, Basic Ergodic Theory, Birkhauser Verlag, 1998.
[3] Peter Walters, An Introduction to Ergodic Theory, Springer.

Singular Homology Theory. Singular complex and homology groups, functorial properties, relative homology groups, the Eilenberg-Steenrod axioms of homology theory. Long exact sequences. The reduced homology groups, the Mayer-Vietoris sequence. Homology of spheres. The degree of self maps of n-sphere, The Brouwer's fixed point theorem, Hairy Ball Theorem, Lusternik-Schnirelmann Theorem, Jordan-Brouwer Separation Theorem, Invariance of Domain.

References:

[1] E H Spanier, Algebraic Topology, Springer Verlag, 1989.
[2] G E Bredon, Topology and geometry, Springer Verlag, 2005.
[3] A Dold, Lectures on Algebraic Topology, Springer-Verlag, Second Edition 1980.
[4] J J Rotman, An Introduction to Algebraic Topology, Springer Verlag, 1988.
[5] M.J. Greenberg and J.R. Harper, Algebraic Topology- A first course, Addison-Wesley Publishing Company, Inc. 1981.
[6] W S Massey, A Basic Course in Algebraic topology, Springer- Verlag, 1991.

Group - C

Graph and Network Theory. Non-Oriented Linear Graphs: Introduction of graphs & networks, Paths & Circuits, Euler Graph, M-Graph, Non-separable graph, Collection of Paths, Traversability: Eulerian Graphs & Hamiltonian Graphs.

Matrix Representation of Linear Graphs & Trees: Incidence Matrix, Tress, Spanning trees, Steiner Trees, Bottleneck Steiner trees, Forests, Branching, Circuits matrix.

Oriented Linear Graphs: Incident & Circuit matrices of Oriented graphs, Elementary tree transformation values of non zero major determinants of a circuit matrix.

Graphs Theory Algorithms, Dijikstra's Algorithm for finding the shortest path in a Network, Double Sweep Algorithm for finding k-shortest paths for a given k. Spanning tree Algorithm, Minimum Spanning Tree Algorithm-Maximum Branching Algorithm.

References.

[1] Mayeda W. : Graph Theory, Wiley-Interscience, John Wiley & Sons, Inc. 1972.
Harary F. : Graph Theory, & Theoretical Physics, Academic Press, 1967.
[2] Evans J.R. & Minieka E, Optimization Algorithms for Networks & Graphs (2nd Edition) Marcel Dekker, 1992.
[3] V. Chachre, Ghere P.M. & Moore J.M.: Applications of Graph Theory Algorithm, Elsevier North Holland, Inc. 1979.
[4] Thulasiraman K. and Swami M.N. S., Graphs: Theory & Algorithms, Wiley Interscience Publication, 1992.
[5] Jungnickel, Dieter, Graphs, Networks and Algorithms (4th edition), Spriner-Verlag, Heidelberg, 2013.

Convex and Nonsmooth Analysis. Convex sets, Convexity-preserving operations for a set, Relative Interior, Asymptotic cone, Separation theorems, Farkas Lemma, Conical approximations of convex sets, Bouligand tangent and normal cones. Convex functions of several variables, Affine functions, Functional operations preserving convexity of function, Infimal convolution, Convex hull and closed convex hull of a function, Continuity properties, Sublinear functions, Support function, Norms and their duals, Polarity. Subdifferential of convex functions, Geometric construction and interpretation, properties of subdifferential, Minimality conditions, Mean-value theorem, Calculus rules with subdifferentials, Subdifferential as a multifunction, monotonicity and continuity properties of the subdifferential, Subdifferential and limits of gradients.

References.

[1] Convex, Analysis and Minimization Algorithms I, Jean-Baptiste Hiriart-Urruty and Claude Lemarechal, Springer- Verlag, Berlin, 1996.
[2] Convex Analysis and Nonlinear Optimization : Theory and Examples, Jonathan M. Borwein Adrian and S. Lewis, CMS Books in Mathematics, Springer Verlag, New York, 2006.
[3] Convex Analysis, R. Tyrrell Rockafellar, Priceton University Press, Princeton, New Jersey, 1997.

Multi-objective Optimization

Multiple Objective Linear Programming Problem, Multiple Criteria Examples, Utility Functions,Non Dominated Criteria Vectors and Efficient Points, Point Estimate Weighted Sums Approach, Optimal Weighting Vectors, Scaling and Reduced Feasible Region Methods, Vector Maximum Algorithm. Formulation of the Multiple Objective Model, Method of Solutions, Augmented Goal Programming, Interactive Multiple Objective Methods. Multiple Objective Linear Fractional Programming. Multiple Objective Non linear Programming Problem, Efficiency and Non- Dominance, Weakly and Strictly Efficient Solutions, Proper Efficiency and Proper Non- Dominance. Weighted Sum Scalarization : (Weak) Efficiency, Proper Efficiency, Optimality Conditions. Scalarization Techniques : The €-Constraint Method, The Hybrid Method, The Elastic Constraint Method and Benson's Method.

References:

[1] Ralph E.Steuer : Multi Criteria Optimization: Theory, Computation, and Application, John Wiley and Sons, 1986. Chapters-1, 6, 7, 8, 9, 12.
[2] James P. Ignizio : Linear Programming in Single and Multiple Objective Systems, Prentice-Hall Inc. , Englewood Cliffs, N.J 1981. Chapters- 16, 17, 20.
[3] Matthias Ehrgott: Multicriteria Optimization, Springer, Berlin, Heidelberg, 2005, Second Edition, Chapters- 2, 3,4.

Combinatorial Mathematics. Permutations and combinations, The Rules of Sum and Product, Distributions of Distinct Objects, Distributions of Nondistinct Objects.

Generating Functions for Combinations, Enumerators for Permutations, Distributions of Distinct Objects into Nondistubct Cells, Partitions of Integers, Elementary Relations.

Recurrence Relations, Linear Recurrence Relations with Constant Coefficients, Solution by the technique of Generating Functions, Recurrence relations with two indices.

The Principle of Inclusion and Exclusion. The General Formula, Derangements, Permutations with Restrictions on Relative positions.

Polya’s Theory of Counting, Equivalence Classes under a Permutation Group, Equivalence Classes of Functions, Weights and Inventories of Functions, Polya’s Fundamental Theorem. Generalization of Polya’s Theorem.

Block designs, Complete block designs, Orthogonal Latin Squares, Balanced Incomplete Block designs. Construction of Block designs.

References.

[1] Introduction to Combinatorial Mathematics by C.L. Ltd (McGraw-Hill), 1968.
[2] An Introduction to Combinatorial Analysis by J. Riordan (John Wiley & Sons), 1958.
[3] R P Grimaldi, Discrete and Combinatorial Mathematics, 4ed, Addision-Wesley, New York, 1998.
[4] S. Barnett, Discrete Mathematics, Numbers and Beyond, Addision-Wesley, Singapore, 1998

Parallel Iterative methods for Partial Differential Equations. Speedup; efficiency; Amdahl's law; point and block parallel relaxation algorithms (Jacobi, Gauss-Seidel, SOR); triangular matrix decomposition; quadrant interlocking factorisation method; red-black ordering; application to elliptic BVPs; parallel ADI algorithms; parallel conjugate-gradient method; parallel multi-grid method; parallel domain decomposition method.

The alternating group explicit method for two point BVPs (natural, derivative, mixed, periodic) and their convergence analysis; the MAGE and NAGE methods; the computational complexity of the AGE method; the Newton-AGE method.

Parabolic equation: AGE algorithm for diffusion-convection equation and its convergence analysis; stability analysis of more general scheme; CAGE method; AGE method for fourth order parabolic equation.

Hyperbolic equation: Group explicit method for first and second order hyperbolic equations; GER, GEL, GAGE, GEU, GEC algorithms; stability analysis of GE method; AGE iterative method for first and second order hyperbolic equations.

Elliptic equation: Douglas-Rachford algorithm; BLAGE iterative algorithm with different boundary conditions; AGE-DG algorithm; parallel implementation.

This course consists of theory paper and computer practical.

References.

[1] Y. Saad, Iterative Methods for Sparse Linear Systems, SIAM, Philadelphia (2003).
[2] L.A. Hageman and D.M. Young, Applied Iterative Methods, Dover publication, New York (2004).
[3] D.M. Young, Iterative Solution of Large Linear Systems, Academic Press, New York (1971).
[4] Jianping Zhu, Solving Partial Differential Equations on Parallel Computers, World Scientific, New Jersey (1994).
[5] D.J. Evans, Group Explicit Methods for the Numerical Solution of Partial Differential Equations, Gordon and Breach Science publisher, Amsterdam (1997).

Hydrodynamic Stability Theory. The concept of hydrodynamic stability, the stability of superposed fluids; the Rayleign-taylor instability-the case of two uniform fluids of constant densities separated by a horizontal boundary, the case of exponentially varying density. The Kelvin – Helmholtz stability.

The stability of coquette flow – Rayleigh's criterian. Analytical discussion of stability of inviscid Couette flow. Oscillations of a rotating column of liquid. Thermal stability Orr-Sommerfeld equation, Rayleigh's theorems.

References.

[1] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Oxford University Press, (1961)

[2] P.G. Draxin and W.H. Reid, Hydodynamic Stability, Cambridge University Press 1981)

Advanced Compressible Flows. One-dimensional gas flow (with perfect and van der Waals gas, gravitation, viscosity, heat addition, and conduction), Diffusion, Shock waves (discontinuity surface, jump condition, strength, thickness, reflection, structure, heat addition and MHD effects), Detonation and Deflagration waves, Methods of solution of compressible flow problems. Dimensional analysis and similarity method, Self-similar motion of spherical, cylindrical and plane waves in a gas. Two dimensional subsonic and supersonic flow with linearized theory, Two dimensional subsonic potential flows(Rayleigh-Janzen Method), Two dimensional supersonic flow with method of characteristics. Anisentropic rotational flow of inviscid compressible fluid.

References.

[1] Similarity and Dimensional Method in Mechanics, L.I.Sedov, Mir Publisher, 1982

[2] Fluid Mechanics, L.D.Landau and E.M.Lifshitz, Pragamon Press, 1989

[3] Introduction to the theory of compressible flow, S.I.Pai, D. Van Nostrand Company, 1958

[4] Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Ya. B. Zel'dovich, and Yu.P. Raizer, Academic Press, 1966.

[5] Introductory Fluid Mechanics, J.Katz , Cambridge University Press, 2010

Elliptic Curves and Cryptography. Finite field arithmetic, Geometry and arithmetic of elliptic curves, torsion points, Elliptic curves over finite fields,Determination of number of points on elliptic curves,Discrete Logarithm Problem, Elliptic curve cryptography - including key agreement and key trans- port, Other applications such as factoring, primality testing, Elliptic curves over Q, Elliptic curves over C, complex multiplication, Divisors, Isogenies, Pairings and cryptography from pairings.

References:

[1] Lawrence C. Washington, Elliptic Curves, Number Theory and Cryptography, CRC Press, 2008.

[2] Darrel Hankerson, Alfred Menezes, Scott Vanstone, Guide to Elliptic curve Cryptog- raphy, Springer, 2004.

[3] Ian F. blake, Gadiel Seroussi, Nigel p. Smart, Advances in Elliptic curve cryptography, London University Press, 2005.

HEAD

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF DELHI
DELHI – 110007

Tel. No. 27666658

Ph.D. Programme in Mathematics
(Effective from 2014-2015 onwards)

ELIGIBILITY

i) A candidate must have obtained a Master's/M.Phil degree of the University of Delhi, or any other recognized University, or any degree recognized as equivalent to Master's/M.Phil degree Mathematics. She/he must have obtained either a minimum of 50% marks or equivalent grading in the M.Phil degree or a minimum of 55% marks or equivalent grading in the Master's degree.

ii) The following categories of candidates can be provisionally registered for the degree of Doctor of Philosophy in Mathematics.

a. Candidates having fellowships/scholarships instituted by the University/national and international agencies under schemes approved/recognized by the University. They will be called for an interview and final selection will be based on the performance in the interview.

b. Candidates who are otherwise eligible for admission to the Ph.D. Programme and do not have any financial assistance, will be awarded UGC (non-NET) Fellowship through an entrance examination to be conducted by the Department. The candidates will be short-listed for admission based on the entrance test. Final selection will be based on the performance in the interview.

c. Foreign students with their national or other fellowships recognized by the University of sponsored by their employers, may be given provisional admission, followed by confirmation through due process after a stipulated period of time.

d. The University/College teachers holding a permanent, temporary or ad hoc positions and having completed two years of service as teacher in a Department/Constituent Colleges of the University of Delhi.

e. Candidates sponsored by their employers shall be considered only if they get study leave for a period of two years to fulfill residency requirements of the University of Delhi. Provided however in order to advance research in strategic areas of national concern, scientist/professionals working at defense and space institutions/organizations of the Government of India/State Government and with whom the University has signed a Memorandum of Under, will be allowed to purse Ph.D. while working in their organizations provided that the DRC recommends that their work in their parent organization is relevant to their Ph.D. research. Such students may also be exempt from the requirement of course work under *amended Clause 4 E of Ordinance VI-B* (amended vide notification dated 29th November, 2013)

f. Permanent teachers/ employees who are in service in any other recognized University/ College/ Research Institute in Indian and have a minimum of three years teaching/ research experience, will be considered if they get study leave for a period of two years to fulfill residency requirements of the University of Delhi.

REGISTRATION. A candidate for Ph.D. Programme in the Department of Mathematics will be registered by the Board of Research Studies (Mathematical Sciences) on the recommendations of the Departmental Research Studies subject to the availability of a supervisor in the area of his/her interest.

COURSE WORK. All candidates, except those who have been exempted from course work by the DRC, will be required to qualify Pre-Ph.D examination with three courses listed hereunder. However, the advisory committee may prescribe a specialized course not listed below in lieu of one of the three. Such a course will be offered by the supervisor(s) after providing the details of the course to the Departmental Research committee; the examination will be held in the same manner as of the other papers.

Group- A

i) Distribution Theory and Calculus on Banach Spaces
ii) Operator Theory and Function Spaces
iii) Geometric Function Theory
iv) Introduction to Operator Algebras
v) Advanced Frame Theory

Group- B

i) Rings and Modules
ii) Group Rings
iii) Advanced Commutative Algebra
iv) Differential Manifolds
v) Topological Dynamics
vi) Topological Structures
vii) Chaos Theory
viii) Ergodic Theory
ix) Singular Homology Theory

Group- C

i) Graph and Network Theory
ii) Convex and Nonsmooth Analysis / Multi-objective Optimization
iii) Combinatorial Mathematics
iv) Parallel Iterative methods for Partial Differential Equations
v) Hydrodynamic Stability Theory
vi) Advanced Compressible Flows
vii) Elliptic Curves and Cryptography

SCHEME OF EVALUATION. The candidates will be evaluated in each course out of seventy five marks. The evaluation in each course will be based on the student's performance in a written examination and internal assessment. Each paper will be of fifty marks and of three hours duration. Twenty five marks in each course are assigned for internal assessment which will be based on assignments, attendance, class-room performance and seminars.

- 50% marks in each course will be required to pass.
- Supplementary examination will be conducted for those who fail in the course work examination. Failed students can appear only in the supplementary examination of the same year which will be conducted within 3 months of the declaration of the result of course work.
- No student shall be allowed to appear in course work examination more than twice.

CRITERION FOR RECOGNIZING SUPERVISORS FROM COLLEGES OF UNIVERSITY OF DELHI.

- A permanent college teacher with at least 3 years of teaching experience and 2 years of research experience after obtaining Ph.D. degree may be recognized as a supervisor for guiding Ph.D. students.

- For recognition, the teacher should be active in research; he/she should have published at least one research paper in journals listed in SCI/MatSciNet/Scopus/any recognized Database from post-Ph.D. work during last five years.

- The number candidates permitted to be registered may be as follows:
  Associate Professor: 4 Assistant Professor: 2

- The above guidelines shall be applicable for new registrations even if the supervisors have already registered students.

FUNCTIONING OF ADVISORY COMMITTEE

- Supervisors as convener of the Advisory Committee must ensure that a meeting of the committee takes place at least once in a year. The candidate may also be asked to make presentation before the committee. Minutes of the meetings should be submitted to the Department. This will be applicable to all registered students.

- All the annual reports of the candidates to be sent to BRS should go through DRC to enable DRC to verify if the meetings of the advisory committees have taken place.

SYLLABUS

Group- A

Distribution Theory and Calculus on Banach Spaces. Test functions and distributions, some operations with distributions, local properties of distributions, convolutions of distributions, tempered distributions and Fourier transform, fundamental solutions.

The Frechet derivative, chain rule and mean value theorems, implicit function theorem, extremum problems and Lagrange multipliers.

References.

[1] W. Cheney : Analysis for Applied Mathematics; Springer -Verlag, 2001.
[2] S. Kesavan : Topics in Functional Analysis and Applications; New Age International Publishers, 2008
[3] W. Rudin : Functional Analysis; Tata Mc-Graw Hill, 1991.
[4] Robert S. Strichartz : A guide to distribution theory and Fourier transforms; World Scientific Publishing Co., 2003.

Operator Theory and Function Spaces . Fredholm operators; semi-Fredholm operators; index of a Fredholm (semi- Fredholm) Operator; essential spectrum; Weyl spectrum and Weyl theorem; direct sums of operators, their spectra and numerical ranges; weighted shifts, their norms and spectral radii; normaloid, convexoid and spectraloid operators.

Invariant subspace problem; transitive, reductive and reflexive algebras; von-Neumann algebras.

Hardy spaces: Poisson's kernel; Fatou's theorem; zero sets of functions; multiplication, composition, Toeplitz and Hankel operators.

References.

[1] Vladimir V.Peller, Hankel operators and their applications, Springer, 2002.
[2] Nikolai L.Vasilevski, Commutative algebras of Toeplitz operators on Bergman space, Birkhauser, 2008.
[3] N.Young, An introduction to Hilbert space, Cambridge University Press, 1988.
[4] P.R.Halmos, A Hilbert space problem book, II Ed., D.Van Nostrand Company, 1982.
[5] H.Radjavi and P.Rosenthal, Invariant subspaces, Springer Verlag, 1973.

Geometric Function Theory. Area theorem, growth, distortion theorems, coefficient estimates for univalent functions special classes of univalent functions. Lowner's theory and its applications; outline of de Banges proof of Bieberbach conjecture. Generalization of the area theorem, Grunsky inequalities, exponentiation of the Grunsky inequalities, Logarithmic coefficients. Subordination and Sharpened form of Schwarz Lemma

References.

[1] P. Duren, Univalent Functions, Springer, New York, 1983
[2] A. W. Goodman, Univalent Functions I & II, Mariner, Florida, 1983
[3] Ch. Pommerenke, Univalent Functions, Van den Hoek and Ruprecht, Göttingen, 1975.
[4] M. Rosenblum, J. Rovnyak, Topics in Hardy Classes and Univalent Functions, Birkhauser Verlag, 1994
[5] D. J. Hallenbeck, T. H. MacGregor, Linear Problems and Convexity Techniques in Geometric Function Theory, Pitman Adv. Publ. Program, Boston-London-Melbourne,1984.
[6] I.Graham, G. Kohr, Geometric Function Theory in One and Higher Dimensions, Marcel Dekker, New York, 2003.

Introduction to Operator Algebras. Basic definitions and examples of Banach*-algebras, Spectrum of a Banach algebra element, L^1-algebras and Beurling algebras, Tensor products of Banach algebras, Multiplicative linear functional, The Gelfand representations, Fourier algebra, Functional calculus of in C*-algebras, Continuity and homomorphisms, Approximate identities in C*-algebras, Quotient algebras of C*-algebras, Representations and positive linear functional, Double Commutation Theorem, Enveloping von Neumann algebra of a C*-algebra, Tensor products of C*-algebras.

References.

[1] J.Diximier, C*-algebras, North-Holland Amersdem, 1977.
[2] R.V. Kadison and J.R.Ringrose, Fundamentals of the theory of operator algebras, Graduate studies in Mathematics, 15, AMS, Providence, 1997.
[3] E.Kaniuth, A course in commutative Banach algebras, Springer Verlag, 2008.
[4] M.Takesaki, Theory of Operator algebras, Springer Verlag, 2001.

Advanced Frame Theory. An overview on frames. B-Spline Symmetric B-Splines. Frames of translates. The canonical dual frame . Compactly supported generators. An application to sam-pling theory. Shift-Invariant Systems, Frame-properties of shift-invariant system, Representations of the frame operator. Gabor Frames in $L^2(R)$. Basic Gabor frame theory, Tight Gabor frames, The duals of a Gabor frame, Explicit construction of dual frame pairs, Popular Gabor conditions Representations of the Gabor frame operator and duality. Wavelet frames in $L^2(R)$.

References.

[1] O. Christensen, Frames and bases (An introductory course), Birkhauser, Boston (2008).
[2] I. Daubechies, Ten Lectures on wavelets, SIAM, Philadelphia (1992).
[3] R. Young, A introduction to non-harmonic Fourier series, Academic Press, New York (revised edition 2001).

Group- B

Rings and Modules.Essential and superfluous submodules, Decomposition of rings, Generating and cogenerating, Modules with composition series, Fitting Lemma, Indecomposable decompositions of modules, Projective modules and generators, Radicals of projective modules, Projective covers, Injective hulls, Cogenerators, Flat modules. Singular submodules, Localization and maximal quotient rings. Essential finite generation, Finite dimensionality, Uniform modules and Goldie rings. Regular rings, Strongly regular rings, Unit regular rings, Right π- regular rings. Baer rings, Rickart rings. Baer*rings, Rickart*rings.

References.

[1] A.F.Anderson and K.R.Fuller: Rings and categories of modules, Springer- Verlag,1991 (Relevant sections of Ch. 2,3,4,5).
[2] S.K.Berberian : Baer Vings,Springer Verlag, New York ,1972 (Ch.1, sections 3, 4).
[3] K.R.Goodeari : Ring theory (Non singular rings and modules), Marcel Dekker,Inc. New York (Relevant sections of Ch. 1,2,3).
[4] K.R.Goodeari : Von Neumann regular rings,Pitman, London, 1979 (Ch. 1,3,4).
[5] T.Y.Lam: Lectures on Modules and rings, Springer Verlag, 1998(Ch. 3 ,section 7(d)).

Group Rings. Twisted Group Rings, Tensor Products, Idempotents, Finite groups, Augmentation annihilators, Group algebra as injective modules, Linear identities. The Center, Finite conjugate groups, Chain conditions.

References.

[1] D. S. Passman The Algebraic structure of Group Rings, Dover Publications (Reprint edition). 2011.
[2] S. K. Sehgal, Topics in Group Rings, Marcel Dekker, New York, and Basel, 1978.
[3] I.B.S. Passi, Group Rings and their Augmentation Ideals Lecture Notes in Mathematics 715, Springer, New York, 1979.
[4] A. A. BOVDI, Group Rings Uzhgorod State University, 1978.
[5] D. S. Passman, Infinite Group Rings, Pure and Applied Math. 6, Marcel Dekkar, New York, 1971.
[6] P. Rihenboim, Rings and Modules, Interscience Tracts in Pure and Applied Mathematics, No.6, Interscience, New York, 1969.
[7] C.P. Milies and S.K. Sehgal, An Introduction to Group Rings, Kluwer Academic Publishers, Dordrecht, 2002.

Advanced Commutative Algebra. Direct limit, Inverse limit, Graded rings and modules, Associated graded rings, I-adic completion, Krull's intersection theorem, Hensel's lemma, Hilbert function, Hilbert polynomial, Dimension theory of Noetherian local rings, Regular local rings, UFD property of regular local rings, Hom functor, Tensor functor, I-torsion functor, Flat modules, Projective and injective modules, Complexes, Projective and injective resolution, Derived functor, Tor and ext functor, Minimal resolution, Regular sequences, Cohen-Macaulay rings and modules.

References:

[1] H. Matsumura, Commutative ring theory, Cambridge university press, 1989.
[2] Balwant Singh, Basic commutative algebra, World scientific publishing co., 2011.
[3] D. Eisenbud, Commutative algebra with a view towards algebraic geometry, Springer verlag, 1995.
[4] M.F. Atiyah & I.G. Macdonald, Introduction to commutative algebra, Addison Wesley, 1969.

Differential Manifolds. The derivative, continuously differentiable functions, the inverse function theorem, the implicit function theorem. Topological manifolds, partitions of unity, imbeddings and immersions, manifolds with boundary, submanifolds. Tangent vectors and differentials, Sard's theorem and regular values, vector fields and flows, tangent bundles, smooth maps and their differentials. Smooth manifolds, smooth manifolds with boundary, smooth sub-manifolds, construction of smooth functions.

References.

[1] G.E. Bredon, Topology and Geometry, Springer-verlag, 1993.
[2] L. Conlon, Differentiable Manifolds, Second Edition, Birkhauser,2003.
[3] A. Kosinski, Differential Manifolds, Academic Press, 1992.
[4] M. Spivak, A Comprehensive Introduction to Differential Geometry, Vol. I; Publish or Perish, 1979.

Topological Dynamics. Dynamical Systems: Definition and examples (including real life examples), Orbits, Types of orbits, Topological conjugancy and orbits, Phase Portrait - Graphical Analysis of orbit, Periodic points and stable sets, Omega and alpha limit sets and their properties, Sarkoviskii's Theorem, Dynamics of Logistic Functions, Shift spaces and subshifts, Subshift of finite type and subshift represented by a matrix.

Definition and examples of expansive homeomorphisms, Properties of expansive homeomorphisms, Non-existence of expansive homeomorphism on the unit interval and unit circle, Generators and weak generators, Generators and expansive homeomorphisms, Converging semiorbits for expansive homeomorphisms, Definition and examples of shadowing property, properties of shadowing property, Topological Stability, Anosov maps and topological stability.

REFERENCE BOOKS:
[1] N. Aoki and K. Hiraide, Topological theory of Dynamical Systems, Recent Advances, North Holland Publications, 1994.
[2] D. Lind and B. Marcus, Symbolic Dynamics and Coding, Cambridge University Press, 1996.
[3] Clark Robinson, Dynamical Systems, Stability, Symbolic Dynamics and Chaos, CRC Press,1998.
[4] J. De. Vries, Elements of Topological Dynamics, Mathematics and its applications, Kluwer Academic Publishers, 2000.
[5] D. Hanselman and B. Littlefiels, Mastering MATLAB, Pearson Education, 2005.
Introduction to Dynamical Systems, Brin and Stuck, Cambridge Univ. Press, 2002

Topological Structures. Dimension Theory: Definition and basic properties of the three dimension function inc. Inc. and dim, Characterization and subset theorems, equality of dim X and dim βX equality of lnd X and lnd βX.

Paracompactness: Paracompactness and full normality, presentation of paracompactness under mappings, Hanai-Moritastone theorem, products of paracompact spaces, countable paracompactness, strong paracompactness characterizations of strong paracompactness in regular spaces, products and subspaces of strongly paracompact spaces, pointwise paracompactness Arens Dugundji theorem, collectionwise normal spaces, Ding’s example of a normal space which is not collectionwise normal.

Bitopological Spaces: Basic concepts, subspaces and products Separation and covering axioms.

References.
[1] R. Engelking: General Topology, Polish Scientific Publishers Warszawa, 2nd Ed., 1977.
[2] K. Nagami: DimensionTheory, Academic Press, New York, 1970.
[3] W.J. Pervin: Foundations of General Topology, Academic Press Inc., New York, 1964.
[4] S. Willard: General Topology, Addison-Wesley Publishing Co. Inc., 1970.

Chaos Theory. Topological transitivity: Examples and properties, Topological mixing: Examples and Properties, Transitivity and limit sets for maps on I , Characterizing topological mixing in terms of topological transitivity for maps on I , Sensitive dependence on initial conditions, Devaney's definition of chaos, Logistic maps and shift maps as chaotic maps, Period three implies chaos, Relation between transitivity and chaos on I .

Topological Entropy: Definitions, Entropy of interval maps, Horseshoes, Entropy of cycles, Continuity properties of the Entropy, Entropy of shift spaces, Entropy for circle maps, Various other definitions of Chaos and their interrelationships.

References.

[1] L. Alseda, J. Llibre, M. Misiurewicz, Combinatorial Dynamics and Entropy inDimension One, Advanced Series in Nonlinear Dynamics, 2000.
[2] L. S. Block and W. A. Coppel, Dynamics in One dimension, Springer, 1992.
[3] R. L. Devaney, A First Course in Chaotic Dynamical Systems, Westview Press, 1992.
[4] D. Hanselman and B. Littlefiels, Mastering MATLAB, Pearson Education,2005.
[5] Clark Robinson, Dynamical Systems, Stability, Symbolic Dynamics and Chaos, CRC press, 1999.
[6] S. Ruette, Chaos for continuous interval maps: A survey of relationship between various sorts of chaos, 2003.
[7] Introduction to Dynamical Systems, Brin and Stuck, Cambridge Univ. Press, 2002

Ergodic Theory. Measure preserving transformations and examples, Recurrence, Poincare's Recurrence theorem, Ergodicity, ergodicity of shift transformations

Ergodic theorems of Birkhoff and Von Neuman, Mixing, Weak-mixing and their characterizations, the isomorphism problem: conjugacy, Spectral equivalence, Transformations with discrete spectrum, Entropy, Kolmogorov-Sinai theorem, K-systems examples of calculation of entropy, Unique ergodicity, uniformly distributed sequences, applications to Diophantine approximation.

References.

[1] P. R. Halmos, Lectures on Ergodic Theory, American Mathematical Society, 2006
[2] M. G. Nadkarni, Basic Ergodic Theory, Birkhauser Verlag, 1998.
[3] Peter Walters, An Introduction to Ergodic Theory, Springer.

Singular Homology Theory. Singular complex and homology groups, functorial properties, relative homology groups, the Eilenberg-Steenrod axioms of homology theory. Long exact sequences. The reduced homology groups, the Mayer-Vietoris sequence. Homology of spheres. The degree of self maps of n-sphere, The Brouwer's fixed point theorem, Hairy Ball Theorem, Lusternik-Schnirelmann Theorem, Jordan-Brouwer Separation Theorem, Invariance of Domain.

References:

[1] E H Spanier, Algebraic Topology, Springer Verlag, 1989.
[2] G E Bredon, Topology and geometry, Springer Verlag, 2005.
[3] A Dold, Lectures on Algebraic Topology, Springer-Verlag, Second Edition 1980.
[4] J J Rotman, An Introduction to Algebraic Topology, Springer Verlag, 1988.
[5] M.J. Greenberg and J.R. Harper, Algebraic Topology- A first course, Addison-Wesley Publishing Company, Inc. 1981.
[6] W S Massey, A Basic Course in Algebraic topology, Springer- Verlag, 1991.

Group - C

Graph and Network Theory. Non-Oriented Linear Graphs: Introduction of graphs & networks, Paths & Circuits, Euler Graph, M-Graph, Non-separable graph, Collection of Paths, Traversability: Eulerian Graphs & Hamiltonian Graphs.

Matrix Representation of Linear Graphs & Trees: Incidence Matrix, Tress, Spanning trees, Steiner Trees, Bottleneck Steiner trees, Forests, Branching, Circuits matrix.

Oriented Linear Graphs: Incident & Circuit matrices of Oriented graphs, Elementary tree transformation values of non zero major determinants of a circuit matrix.

Graphs Theory Algorithms, Dijikstra's Algorithm for finding the shortest path in a Network, Double Sweep Algorithm for finding k-shortest paths for a given k. Spanning tree Algorithm, Minimum Spanning Tree Algorithm-Maximum Branching Algorithm.

References.

[1] Mayeda W. : Graph Theory, Wiley-Interscience, John Wiley & Sons, Inc. 1972.
Harary F. : Graph Theory, & Theoretical Physics, Academic Press, 1967.
[2] Evans J.R. & Minieka E, Optimization Algorithms for Networks & Graphs (2nd Edition) Marcel Dekker, 1992.
[3] V. Chachre, Ghere P.M. & Moore J.M.: Applications of Graph Theory Algorithm, Elsevier North Holland, Inc. 1979.
[4] Thulasiraman K. and Swami M.N. S., Graphs: Theory & Algorithms, Wiley Interscience Publication, 1992.
[5] Jungnickel, Dieter, Graphs, Networks and Algorithms (4th edition), Spriner-Verlag, Heidelberg, 2013.

Convex and Nonsmooth Analysis. Convex sets, Convexity-preserving operations for a set, Relative Interior, Asymptotic cone, Separation theorems, Farkas Lemma, Conical approximations of convex sets, Bouligand tangent and normal cones. Convex functions of several variables, Affine functions, Functional operations preserving convexity of function, Infimal convolution, Convex hull and closed convex hull of a function, Continuity properties, Sublinear functions, Support function, Norms and their duals, Polarity. Subdifferential of convex functions, Geometric construction and interpretation, properties of subdifferential, Minimality conditions, Mean-value theorem, Calculus rules with subdifferentials, Subdifferential as a multifunction, monotonicity and continuity properties of the subdifferential, Subdifferential and limits of gradients.

References.

[1] Convex, Analysis and Minimization Algorithms I, Jean-Baptiste Hiriart-Urruty and Claude Lemarechal, Springer- Verlag, Berlin, 1996.
[2] Convex Analysis and Nonlinear Optimization : Theory and Examples, Jonathan M. Borwein Adrian and S. Lewis, CMS Books in Mathematics, Springer Verlag, New York, 2006.
[3] Convex Analysis, R. Tyrrell Rockafellar, Priceton University Press, Princeton, New Jersey, 1997.

Multi-objective Optimization

Multiple Objective Linear Programming Problem, Multiple Criteria Examples, Utility Functions,Non Dominated Criteria Vectors and Efficient Points, Point Estimate Weighted Sums Approach, Optimal Weighting Vectors, Scaling and Reduced Feasible Region Methods, Vector Maximum Algorithm. Formulation of the Multiple Objective Model, Method of Solutions, Augmented Goal Programming, Interactive Multiple Objective Methods. Multiple Objective Linear Fractional Programming. Multiple Objective Non linear Programming Problem, Efficiency and Non- Dominance, Weakly and Strictly Efficient Solutions, Proper Efficiency and Proper Non- Dominance. Weighted Sum Scalarization : (Weak) Efficiency, Proper Efficiency, Optimality Conditions. Scalarization Techniques : The €-Constraint Method, The Hybrid Method, The Elastic Constraint Method and Benson's Method.

References:

[1] Ralph E.Steuer : Multi Criteria Optimization: Theory, Computation, and Application, John Wiley and Sons, 1986. Chapters-1, 6, 7, 8, 9, 12.
[2] James P. Ignizio : Linear Programming in Single and Multiple Objective Systems, Prentice-Hall Inc. , Englewood Cliffs, N.J 1981. Chapters- 16, 17, 20.
[3] Matthias Ehrgott: Multicriteria Optimization, Springer, Berlin, Heidelberg, 2005, Second Edition, Chapters- 2, 3,4.

Combinatorial Mathematics. Permutations and combinations, The Rules of Sum and Product, Distributions of Distinct Objects, Distributions of Nondistinct Objects.

Generating Functions for Combinations, Enumerators for Permutations, Distributions of Distinct Objects into Nondistubct Cells, Partitions of Integers, Elementary Relations.

Recurrence Relations, Linear Recurrence Relations with Constant Coefficients, Solution by the technique of Generating Functions, Recurrence relations with two indices.

The Principle of Inclusion and Exclusion. The General Formula, Derangements, Permutations with Restrictions on Relative positions.

Polya's Theory of Counting, Equivalence Classes under a Permutation Group, Equivalence Classes of Functions, Weights and Inventories of Functions, Polya's Fundamental Theorem. Generalization of Polya's Theorem.

Block designs, Complete block designs, Orthogonal Latin Squares, Balanced Incomplete Block designs. Construction of Block designs.

References.

[1] Introduction to Combinatorial Mathematics by C.L. Ltd (McGraw-Hill), 1968.
[2] An Introduction to Combinatorial Analysis by J. Riordan (John Wiley & Sons), 1958.
[3] R P Grimaldi, Discrete and Combinatorial Mathematics, 4ed, Addision-Wesley, New York, 1998.
[4] S. Barnett, Discrete Mathematics, Numbers and Beyond, Addision-Wesley, Singapore, 1998

Parallel Iterative methods for Partial Differential Equations. Speedup; efficiency; Amdahl's law; point and block parallel relaxation algorithms (Jacobi, Gauss-Seidel, SOR); triangular matrix decomposition; quadrant interlocking factorisation method; red-black ordering; application to elliptic BVPs; parallel ADI algorithms; parallel conjugate-gradient method; parallel multi-grid method; parallel domain decomposition method.

The alternating group explicit method for two point BVPs (natural, derivative, mixed, periodic) and their convergence analysis; the MAGE and NAGE methods; the computational complexity of the AGE method; the Newton-AGE method.

Parabolic equation: AGE algorithm for diffusion-convection equation and its convergence analysis; stability analysis of more general scheme; CAGE method; AGE method for fourth order parabolic equation.

Hyperbolic equation: Group explicit method for first and second order hyperbolic equations; GER, GEL, GAGE, GEU, GEC algorithms; stability analysis of GE method; AGE iterative method for first and second order hyperbolic equations.

Elliptic equation: Douglas-Rachford algorithm; BLAGE iterative algorithm with different boundary conditions; AGE-DG algorithm; parallel implementation.

This course consists of theory paper and computer practical.

References.

[1] Y. Saad, Iterative Methods for Sparse Linear Systems, SIAM, Philadelphia (2003).
[2] L.A. Hageman and D.M. Young, Applied Iterative Methods, Dover publication, New York (2004).
[3] D.M. Young, Iterative Solution of Large Linear Systems, Academic Press, New York (1971).
[4] Jianping Zhu, Solving Partial Differential Equations on Parallel Computers, World Scientific, New Jersey (1994).
[5] D.J. Evans, Group Explicit Methods for the Numerical Solution of Partial Differential Equations, Gordon and Breach Science publisher, Amsterdam (1997).

Hydrodynamic Stability Theory. The concept of hydrodynamic stability, the stability of superposed fluids; the Rayleign-taylor instability-the case of two uniform fluids of constant densities separated by a horizontal boundary, the case of exponentially varying density. The Kelvin – Helmholtz stability.

The stability of coquette flow – Rayleigh's criterian. Analytical discussion of stability of inviscid Couette flow. Oscillations of a rotating column of liquid. Thermal stability Orr-Sommerfeld equation, Rayleigh's theorems.

References.

[1] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Oxford University Press, (1961)
[2] P.G. Draxin and W.H. Reid, Hydodynamic Stability, Cambridge University Press 1981)

Advanced Compressible Flows. One-dimensional gas flow (with perfect and van der Waals gas, gravitation, viscosity, heat addition, and conduction), Diffusion, Shock waves (discontinuity surface, jump condition, strength, thickness, reflection, structure, heat addition and MHD effects), Detonation and Deflagration waves, Methods of solution of compressible flow problems. Dimensional analysis and similarity method, Self-similar motion of spherical, cylindrical and plane waves in a gas. Two dimensional subsonic and supersonic flow with linearized theory, Two dimensional subsonic potential flows(Rayleigh-Janzen Method), Two dimensional supersonic flow with method of characteristics. Anisentropic rotational flow of inviscid compressible fluid.

References.

[1] Similarity and Dimensional Method in Mechanics, L.I.Sedov, Mir Publisher, 1982
[2] Fluid Mechanics, L.D.Landau and E.M.Lifshitz, Pragamon Press, 1989
[3] Introduction to the theory of compressible flow, S.I.Pai, D. Van Nostrand Company, 1958
[4] Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Ya. B. Zel'dovich, and Yu.P. Raizer, Academic Press, 1966.
[5] Introductory Fluid Mechanics, J.Katz , Cambridge University Press, 2010

Elliptic Curves and Cryptography. Finite field arithmetic, Geometry and arithmetic of elliptic curves, torsion points, Elliptic curves over finite fields,Determination of number of points on elliptic curves,Discrete Logarithm Problem, Elliptic curve cryptography - including key agreement and key trans- port, Other applications such as factoring, primality testing, Elliptic curves over Q, Elliptic curves over C, complex multiplication, Divisors, Isogenies, Pairings and cryptography from pairings.

References:

[1] Lawrence C. Washington, Elliptic Curves, Number Theory and Cryptography, CRC Press, 2008.
[2] Darrel Hankerson, Alfred Menezes, Scott Vanstone, Guide to Elliptic curve Cryptog- raphy, Springer, 2004.
[3] Ian F. blake, Gadiel Seroussi, Nigel p. Smart, Advances in Elliptic curve cryptography, London University Press, 2005.

HEAD

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF DELHI
DELHI – 110007

Tel. No. 27666658

M.Phil (Full-time) Programme in Mathematics
(Effective from 2015-2016 onwards)

ELIGIBILITY. The candidate should have good academic record with first or high second class Master's Degree or an equivalent degree of a foreign University in the subject concerned, or an allied subject to be approved by the Vice-Chancellor on the recommendation of the Head of the Department and the Dean of the Faculty concerned. Relaxation to the reserved category students will be provided as per University rules.

ADMISSION PROCEDURE. Admission to the M.Phil programme will be done on the basis of the relative merit of student's performance at post-graduate examination and an entrance test to be conducted by the Department. The merit list will be prepared by taking into account 25% of marks scored in post-graduate examination and 75% of marks scored in the entrance test.

ALLOCATION OF SUPERVISOR. The allocation of the supervisor for a selected student shall be decided by the M.Phil Committee in a formal manner depending on the number of student per faculty member, the available specialization among the faculty supervisors, and the research interest of the student as indicated during counseling by the student. The allotment/allocation of supervisor shall not be left to the individual student or teacher.

PROGRAMME DESCRIPTION. The M.Phil Programme shall consist of two parts: Part I: Course work and Part II: Dissertation.

Part-I: Course Work. A student shall be required to study four courses selecting at least one from different group listed below:

Group- A

- i) Distribution Theory and Calculus on Banach Spaces
- ii) Operator Theory and Function Spaces
- iv) Introduction to Operator Algebras
- v) Advanced Frame Theory

Group- B

- i) Rings and Modules
- iv) Differential Manifolds
- v) Topological Dynamics
- ix) Singular Homology Theory

Group- C

ii) Convex and Nonsmooth Analysis / Multi-objective Optimization
iii) Combinatorial Mathematics
vi) Advanced Compressible Flows

The candidates will be examined in their coursework out of 300 marks and each course will carry 75 marks. Each course will be of one semester duration with 3 lectures per week.

Part II: Dissertation. After passing the examination in the courses as above, a student shall be required to write a dissertation on a subject approved by the Advisory Committee under the supervision of the Supervisor(s) appointed for the purpose. A student who has secured 50% marks in at least two courses of the Part I Examination may be allowed to proceed for his dissertation work. Such a student shall be permitted to submit his dissertation only when he/she has passed the examination in all the courses prescribed under Part I.

The Dissertation may include results of original research, a fresh interpretation of existing facts and data or a review article of a critical nature or may take such other form as may be determined by the Advisory Committee. The dissertation will be submitted only when the Supervisor(s) concerned is/are satisfied that the Dissertation is worthy of consideration in partial fulfillment of the M.Phil. Degree.

Title of the dissertation should be approved and the appointment of external examiner be made before the submission of the dissertation. The application for approval of the title must include a synopsis together with a list of selected main references.

DURATION. The duration of the Programme will be one and half years and the total span period is 36 months. There shall be no provision for extension of the programme duration on a case to case basis.

The dissertation can only be submitted after one year from the date of admission to the M. Phil programme subject to qualifying Part–I examination by a candidate.

No student shall be allowed to take up any assignment outside the University Department during the programme or before submission of his/her dissertation whichever is earlier.

ATTENDANCE. The minimum percentage of lectures to be attended and seminars to be participated in by the students shall be 2/3 of the lectures delivered and seminars held separately.

SCHEME OF EXAMINATION

(a) The Evaluation in each course will be based on the student's performance in written end-semester examination and internal assessment. The written examination of three hours duration will comprise of 50 marks. The internal assessment on the basis of assignment, attendance, class-room performance and seminars will be of 25 marks.

(b) Supplementary examination will be conducted for those who fail in the Part-I examination and it will be conducted within three months of the declaration of the result of Part-I examination. Students can appear in the supplementary examination only in the papers in which they fail or fail to appear. No student shall be allowed to avail more than two chances in any course of the Part-I examination.

(c) No student shall be allowed to reappear in any course of Part-I examination just to improve upon the score.

(d) The dissertation shall be evaluated by the supervisor(s) and one external examiner to be appointed by the M.Phil Committee.

(e) The total marks for Part – II examination is 200. The weightage of written dissertation will be 150 marks. Both the examiners will be required to submit marks out of 75 separately to the Head of the Department before fixing the date for viva-voce. The remaining 50 marks for the viva-voce shall be awarded jointly by both the examiners.

(f) The candidate shall submit the final dissertation after making corrections/alterations (if any) suggested by the examiners within one month of the viva-voce.

RESULT. The minimum marks required to pass the M.Phil. examination shall be 50% marks in each course of Part-I examination and 50% of marks in Part II separately. The successful candidates shall be classified as follows:

I Division with Distinction:	At least 75% marks in the aggregate.
I Division:	At least 60% marks in the aggregate but below 75% marks
Pass:	All others.

SYLLABUS

Group- A

Distribution Theory and Calculus on Banach Spaces. Test functions and distributions, some operations with distributions, local properties of distributions, convolutions of distributions, tempered distributions and Fourier transform, fundamental solutions.

The Frechet derivative, chain rule and mean value theorems, implicit function theorem, extremum problems and Lagrange multipliers.

References.

[1] W. Cheney : Analysis for Applied Mathematics; Springer -Verlag, 2001.
[2] S. Kesavan : Topics in Functional Analysis and Applications; New Age International Publishers, 2008
[3] W. Rudin : Functional Analysis; Tata Mc-Graw Hill, 1991.
[4] Robert S. Strichartz : A guide to distribution theory and Fourier transforms; World Scientific Publishing Co., 2003.

Operator Theory and Function Spaces . Fredholm operators; semi-Fredholm operators; index of a Fredholm (semi- Fredholm) Operator; essential spectrum; Weyl spectrum and Weyl theorem; direct sums of operators, their spectra and numerical ranges; weighted shifts, their norms and spectral radii; normaloid, convexoid and spectraloid operators.

Invariant subspace problem; transitive, reductive and reflexive algebras; von-Neumann algebras.

Hardy spaces: Poisson’s kernel; Fatou’s theorem; zero sets of functions; multiplication, composition, Toeplitz and Hankel operators.

References.

[1] Vladimir V.Peller, Hankel operators and their applications, Springer, 2002.
[2] Nikolai L.Vasilevski, Commutative algebras of Toeplitz operators on Bergman space, Birkhauser, 2008.
[3] N.Young, An introduction to Hilbert space, Cambridge University Press, 1988.
[4] P.R.Halmos, A Hilbert space problem book, II Ed., D.Van Nostrand Company, 1982.
[5] H.Radjavi and P.Rosenthal, Invariant subspaces, Springer Verlag, 1973.

Introduction to Operator Algebras. Basic definitions and examples of Banach*-algebras, Spectrum of a Banach algebra element, L^1-algebras and Beurling algebras, Tensor products of Banach algebras, Multiplicative linear functional, The Gelfand representations, Fourier algebra, Functional calculus of in C*-algebras, Continuity and homomorphisms, Approximate identities in C*-algebras, Quotient algebras of C*-algebras, Representations and positive linear functional, Double Commutation Theorem, Enveloping von Neumann algebra of a C*-algebra, Tensor products of C*-algebras.

References.

[1] J.Diximier, C*-algebras, North-Holland Amersdem, 1977.
[2] R.V. Kadison and J.R.Ringrose, Fundamentals of the theory of operator algebras, Graduate studies in Mathematics, 15, AMS, Providence, 1997.
[3] E.Kaniuth, A course in commutative Banach algebras, Springer Verlag, 2008.
[4] M.Takesaki, Theory of Operator algebras, Springer Verlag, 2001.

Advanced Frame Theory. An overview on frames. B-Spline Symmetric B-Splines. Frames of translates. The canonical dual frame . Compactly supported generators. An application to sam-pling theory. Shift-Invariant Systems, Frame-properties of shift-invariant system, Representations of the frame operator. Gabor Frames in $L^2(R)$. Basic Gabor frame theory, Tight Gabor frames, The duals of a Gabor frame, Explicit construction of dual frame pairs, Popular Gabor conditions Representations of the Gabor frame operator and duality. Wavelet frames in $L^2(R)$.

References.

[1] O. Christensen, Frames and bases (An introductory course), Birkhauser, Boston (2008).
[2] I. Daubechies, Ten Lectures on wavelets, SIAM, Philadelphia (1992).
[3] R. Young, A introduction to non-harmonic Fourier series, Academic Press, New York (revised edition 2001).

Group- B

Rings and Modules.Essential and superfluous submodules, Decomposition of rings, Generating and cogenerating, Modules with composition series, Fitting Lemma, Indecomposable decompositions of modules, Projective modules and generators, Radicals of projective modules, Projective covers, Injective hulls, Cogenerators, Flat modules. Singular submodules, Localization and maximal quotient rings. Essential finite generation, Finite dimensionality, Uniform modules and Goldie rings. Regular rings, Strongly regular rings, Unit regular rings, Right π- regular rings. Baer rings, Rickart rings. Baer*rings, Rickart*rings.

References.
[1] A.F.Anderson and K.R.Fuller: Rings and categories of modules, Springer- Verlag,1991 (Relevant sections of Ch. 2,3,4,5).
[2] S.K.Berberian : Baer Vings,Springer Verlag, New York ,1972 (Ch.1, sections 3, 4).
[3] K.R.Goodeari : Ring theory (Non singular rings and modules), Marcel Dekker,Inc. New York (Relevant sections of Ch. 1,2,3).
[4] K.R.Goodeari : Von Neumann regular rings,Pitman, London, 1979 (Ch. 1,3,4).
[5] T.Y.Lam: Lectures on Modules and rings, Springer Verlag, 1998(Ch. 3 ,section 7(d)).

Differential Manifolds. The derivative, continuously differentiable functions, the inverse function theorem, the implicit function theorem. Topological manifolds, partitions of unity, imbeddings and immersions, manifolds with boundary, submanifolds. Tangent vectors and differentials, Sard's theorem and regular values, vector fields and flows, tangent bundles, smooth maps and their differentials. Smooth manifolds, smooth manifolds with boundary, smooth sub-manifolds, construction of smooth functions.

References.

[1] G.E. Bredon, Topology and Geometry, Springer-verlag, 1993.
[2] L. Conlon, Differentiable Manifolds, Second Edition, Birkhauser,2003.
[3] A. Kosinski, Differential Manifolds, Academic Press, 1992.
[4] M. Spivak, A Comprehensive Introduction to Differential Geometry, Vol. I; Publish or Perish, 1979.

Topological Dynamics. Dynamical Systems: Definition and examples (including real life examples), Orbits, Types of orbits, Topological conjugancy and orbits, Phase Portrait - Graphical Analysis of orbit, Periodic points and stable sets, Omega and alpha limit sets and their properties, Sarkoviskii's Theorem, Dynamics of Logistic Functions, Shift spaces and subshifts, Subshift of finite type and subshift represented by a matrix.

Definition and examples of expansive homeomorphisms, Properties of expansive homeomorphisms, Non-existence of expansive homeomorphism on the unit interval and unit circle, Generators and weak generators, Generators and expansive homeomorphisms, Converging semiorbits for expansive homeomorphisms, Definition and examples of shadowing property, properties of shadowing property, Topological Stability, Anosov maps and topological stability.

REFERENCE BOOKS:

[1] N. Aoki and K. Hiraide, Topological theory of Dynamical Systems, Recent Advances, North Holland Publications, 1994.
[2] D. Lind and B. Marcus, Symbolic Dynamics and Coding, Cambridge University Press, 1996.
[3] Clark Robinson, Dynamical Systems, Stability, Symbolic Dynamics and Chaos, CRC Press,1998.
[4] J. De. Vries, Elements of Topological Dynamics, Mathematics and its applications, Kluwer Academic Publishers, 2000.
[5] D. Hanselman and B. Littlefiels, Mastering MATLAB, Pearson Education, 2005.
Introduction to Dynamical Systems, Brin and Stuck, Cambridge Univ. Press, 2002

Singular Homology Theory. Singular complex and homology groups, functorial properties, relative homology groups, the Eilenberg-Steenrod axioms of homology theory. Long exact sequences. The reduced homology groups, the Mayer-Vietoris sequence. Homology of spheres. The degree of self maps of n-sphere, The Brouwer's fixed point theorem, Hairy Ball Theorem, Lusternik-Schnirelmann Theorem, Jordan-Brouwer Separation Theorem, Invariance of Domain.

References:

[1] E H Spanier, Algebraic Topology, Springer Verlag, 1989.
[2] G E Bredon, Topology and geometry, Springer Verlag, 2005.
[3] A Dold, Lectures on Algebraic Topology, Springer-Verlag, Second Edition 1980.
[4] J J Rotman, An Introduction to Algebraic Topology, Springer Verlag, 1988.
[5] M.J. Greenberg and J.R. Harper, Algebraic Topology- A first course, Addison-Wesley Publishing Company, Inc. 1981.
[6] W S Massey, A Basic Course in Algebraic topology, Springer- Verlag, 1991.

Group - C

Convex and Nonsmooth Analysis. Convex sets, Convexity-preserving operations for a set, Relative Interior, Asymptotic cone, Separation theorems, Farkas Lemma, Conical approximations of convex sets, Bouligand tangent and normal cones. Convex functions of several variables, Affine functions, Functional operations preserving convexity of function, Infimal convolution, Convex hull and closed convex hull of a function, Continuity properties, Sublinear functions, Support function, Norms and their duals, Polarity. Subdifferential of convex functions, Geometric construction and interpretation, properties of subdifferential, Minimality conditions, Mean-value theorem, Calculus rules with subdifferentials, Subdifferential as a multifunction, monotonicity and continuity properties of the subdifferential, Subdifferential and limits of gradients.

References.

[1] Convex, Analysis and Minimization Algorithms I, Jean-Baptiste Hiriart-Urruty and Claude Lemarechal, Springer- Verlag, Berlin, 1996.
[2] Convex Analysis and Nonlinear Optimization : Theory and Examples, Jonathan M. Borwein Adrian and S. Lewis, CMS Books in Mathematics, Springer Verlag, New York, 2006.
[3] Convex Analysis, R. Tyrrell Rockafellar, Priceton University Press, Princeton, New Jersey, 1997.

Multi-objective Optimization

Multiple Objective Linear Programming Problem, Multiple Criteria Examples, Utility Functions,Non Dominated Criteria Vectors and Efficient Points, Point Estimate Weighted Sums Approach, Optimal Weighting Vectors, Scaling and Reduced Feasible Region Methods, Vector Maximum Algorithm. Formulation of the Multiple Objective Model, Method of Solutions, Augmented Goal Programming, Interactive Multiple Objective Methods. Multiple Objective Linear Fractional Programming. Multiple Objective Non linear Programming Problem, Efficiency and Non- Dominance, Weakly and Strictly Efficient Solutions, Proper Efficiency and Proper Non- Dominance. Weighted Sum Scalarization : (Weak) Efficiency, Proper Efficiency, Optimality Conditions. Scalarization Techniques : The €-Constraint Method, The Hybrid Method, The Elastic Constraint Method and Benson's Method.

References:

[1] Ralph E.Steuer : Multi Criteria Optimization: Theory, Computation, and Application, John Wiley and Sons, 1986. Chapters-1, 6, 7, 8, 9, 12.
[2] James P. Ignizio : Linear Programming in Single and Multiple Objective Systems, Prentice-Hall Inc. , Englewood Cliffs, N.J 1981. Chapters- 16, 17, 20.
[3] Matthias Ehrgott: Multicriteria Optimization, Springer, Berlin, Heidelberg, 2005, Second Edition, Chapters- 2, 3,4.

Combinatorial Mathematics. Permutations and combinations, The Rules of Sum and Product, Distributions of Distinct Objects, Distributions of Nondistinct Objects.

Generating Functions for Combinations, Enumerators for Permutations, Distributions of Distinct Objects into Nondistubct Cells, Partitions of Integers, Elementary Relations.

Recurrence Relations, Linear Recurrence Relations with Constant Coefficients, Solution by the technique of Generating Functions, Recurrence relations with two indices.

The Principle of Inclusion and Exclusion. The General Formula, Derangements, Permutations with Restrictions on Relative positions.

Polya's Theory of Counting, Equivalence Classes under a Permutation Group, Equivalence Classes of Functions, Weights and Inventories of Functions, Polya's Fundamental Theorem. Generalization of Polya's Theorem.

Block designs, Complete block designs, Orthogonal Latin Squares, Balanced Incomplete Block designs. Construction of Block designs.

References.

[1] Introduction to Combinatorial Mathematics by C.L. Ltd (McGraw-Hill), 1968.
[2] An Introduction to Combinatorial Analysis by J. Riordan (John Wiley & Sons), 1958.
[3] R P Grimaldi, Discrete and Combinatorial Mathematics, 4ed, Addision-Wesley, New York, 1998.
[4] S. Barnett, Discrete Mathematics, Numbers and Beyond, Addision-Wesley, Singapore, 1998

Advanced Compressible Flows. One-dimensional gas flow (with perfect and van der Waals gas, gravitation, viscosity, heat addition, and conduction), Diffusion, Shock waves (discontinuity surface, jump condition, strength, thickness, reflection, structure, heat addition and MHD effects), Detonation and Deflagration waves, Methods of solution of compressible flow problems. Dimensional analysis and similarity method, Self-similar motion of spherical, cylindrical and plane waves in a gas. Two dimensional subsonic and supersonic flow with linearized theory, Two dimensional subsonic potential flows(Rayleigh-Janzen Method), Two dimensional supersonic flow with method of characteristics. Anisentropic rotational flow of inviscid compressible fluid.

References.

[1] Similarity and Dimensional Method in Mechanics, L.I.Sedov, Mir Publisher, 1982
[2] Fluid Mechanics, L.D.Landau and E.M.Lifshitz, Pragamon Press, 1989
[3] Introduction to the theory of compressible flow, S.I.Pai, D. Van Nostrand Company, 1958
[4] Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Ya. B. Zel'dovich, and Yu.P. Raizer, Academic Press, 1966.
[5] Introductory Fluid Mechanics, J.Katz , Cambridge University Press, 2010

HEAD

Syllabus for Ph.D Programme

Pre-Ph.D Course Work for the year 2015-16

Paper (i) – Research Methodology (Compulsory Paper)

Paper (ii) – Reading Courses (RC) (Two papers to be chosen)

Research Methodology in Mathematics

Scientific research and literature survey. History of mathematics, finding and solving research problems, role of a supervisor, survey of a research topic, publishing a paper, reviewing a paper, research grant proposal writing, copyright issues, ethics and plagiarism.

Research tools. Searching google (query modifiers), MathSciNet, ZMATH, Scopus, ISI Web of Science, Impact factor, h-index, Google Scholar, ORCID, JStor, Online and open access journals, Virtual
library of various countries

Scientific writing and presentation. Writing a research paper, survey article, thesis writing; LaTeX, PSTricks, Beamer, HTML and MathJaX

Software for Mathematics. Mathematica/Matlab/Scilab/GAP

Reference:

[1] J. Stillwell, Mathematics and its History, Springer International Edition, 4th Indian Reprint, 2005

[2] L. Lamport, LaTeX, a Document Preparation System, 2nd ed, Addison-Wesley, 1994.

[3] Norman E. Steenrod, Paul R. Halmos, Menahem M. Schiffer, Jean A. Dieudonne, How to Write Mathematics, American Mathematical Society, 1973.

[4] Nicholas J. Higham, Handbook of Writing for the Mathematical Sciences, Second Edition, SIAM, 1998.

[5] Donald E. Knuth, Tracy L. Larrabee, and Paul M. Roberts, Mathematical Writing, Mathematical Association of America Washington, D.C., 1989.

[6] Frank Mittelbach, Michel Goossens, Johannes Braams, David Carlisle, Chris Rowley, The LaTeX Companion, 2nd edition (TTCT series), Addison-Wesley, 2004.

[7] Michel Goossens, Frank Mittelbach, Sebastian Rahtz, Denis Roegel, Herbert Voss, The LaTeX Graphics Companion, 2nd edition (TTCT series), Addison-Wesley, 2004

[8] Mathtools documentation (http://mirrors.ctan.org/macros/latex/contrib/mathtools/mathtools.pdf)

[9] Pstricks documentation (http://tug.org/PSTricks/main.cgi?file=doc/docs)

[10] MathJax documentation (http://tug.org/PSTricks/main.cgi?file=doc/docs)

RC(i) - Lie Groups and Lie Algebras

Unit -I : Differential Manifolds Topological manifolds, Charts, Atlases and smooth structure, Smooth maps and diffeomorphism, Partitions of Unity, Tangent space, Tangent map, Vector fields and 1-forms.

Unit -II : Lie Groups Definition and examples, Linear Lie groups, Lie group homomorphism, Lie algebra and the exponential map, Adjoint representation, Homogeneous spaces, Baker-Campbell-Housdorff formula.

Unit -III : Lie Algebras Definition and examples, Classical Lie algebras, Solvable and nilpotent Lie algebras, Lie and Engel theorems, Semisimple and reductive algebras, Semisimplicity of Classical Lie algebras, Killing form and Cartan criterion, Cartan subalgebra, root decomposition and root systems, Weyl group and Weyl chambers, Dynkin diagrams, Classsification of simple Lie algebras.

Unit -IV : Partial Differential Equations on Manifolds Partial differential operators and formal adjoints, Sobolev spaces in R^n, Elliptic estimates in R^n , Elliptic regularity, Fredholm theory and spectral theory of Laplacian.

Suggested Texts:

1. S. Kumaresan, Differential Geometry and Lie Groups, Hindustan Book Agency.

2. Alexander Kirillov Jr, An Introduction to Lie Groups and Lie Algebras, Cambridge University Press.

3. James Humphreys, Introduction to Lie Algebras and Representation Theory, Springer.

4. Brian Hall, Lie Groups, Lie Lagebras, and Representations: An Elementary Introduction, Second Edition, Springer.

5. J. M. Lee, Manifolds and Differential Geometry, Graduate Studies in Mathematics vol 107, AMS.

6. Liviu I Nicolaescu, Lectures on Geometry of Manifolds, Second Edition, World Scientific.

7. A. C. Pipkin, A Course on Integral Equations, Text in Applied Mathematics Series, Springer.

RC(ii) – Representation of Nilpotent Lie Group

Basic facts about Lie groups and Lie algebras, Nilpotent Lie groups, Coadjoint orbits and the dual of g, Some generalities on representations, Elements of kirillov theory, Proof of basic theorems, subgroups of condimension 1 and representations.

References

1. L.J. Corwin and F.D. Grenleaf, Representations of nilpotent Lie groups and their applications, Cambridge University Press, 1990
2. V.S. Varadarajan, Lie groups, Lie algebras and their representations, Prentice-Hall, 1974.

RC(iii) - Univalent Functions

Univalent functions, area theorems, Bieberbach theorem and its applications, subclasses of starlike and convex functions and their generalizations, functions with positive real part, typically real functions.

Close-to-convex functions and the functions of bounded boundary rotation, bounded functions, radius problems and Koebe domains, combination and convolutions of univalent functions, Integrals and integral inequalities, meromorphic functions.

References.

[1]] A. W. Goodman, Univalent Functions I & II, Mariner, Florida, 1983.

[2] P. Duren, Univalent Functions, Springer, New York, 1983

[3] Ch. Pommerenke, Univalent Functions, Van den Hoek and Ruprecht, Göttingen, 1975.

RC(iv) - Theory of Differential Subordination

Jack-Miller-Mocanu Lemma, Admissible functions and fundamental theorems, open door lemma and integral existence theorem, first order differential subordination, Briot-Bouquet differential subordinations, and its generalizations and applications, integral operator, subordination preserving integral operators .

Second order differential subordinations, integral operators preserving functions with positive real part, bounded functions, averaging operators, Hypergemetric functions, Schwarzian derivative, applications to starlikeness and convexity.

References

[1] S. S. Miller and P. T. Mocanu, Differential Subordinations. Theory and. Applications, Marcel Dekker Inc., New York, Basel, 2000.

[2] T. Bulboacˇa, Differential Subordinations and Superordination: Recent Results, Cluj-Napoca,2005.

RC(v) - Harmonic Mappings in the Plane

Harmonic mappings, Argument principle, Dirichlet problem, critical points of harmonic mappings, Lewy's theorem, Heinz's theorem, Rado's theorem

The Rado-Kneser-Choquet theorem, Shear construction, structure of convex mappings, covering theorems and coefficient bounds

Harmonic self mappings of the disk, normalization and normality of harmonic univalent functions, Harmonic Koebe functions and coefficient conjectures, extremal problems, typically real and starlike functions, problems and conjectures in planar harmonic mappings.

Text:

[1] P.L. Duren, Harmonic Mappings in the Plane, Cambridge Univ. Press, Cambridge, 2004.

[2] D. Bshouty and A. Lyzzaik, Problems and Conjectures in Planar Harmonic Mappings, J. Analysis,

Volume 18 (2010), 69–81.

RC (vi) - Operator Spaces

Operator Spaces (concrete and abstract), Completely bounded maps, subspaces, quotients, products, Dual spaces, conjugates, mapping spaces, opposite, representation theorem, The min and max quantizations, Arveson-Wittstock theorem, Column and Row Hilbert spaces. Projective tensor product, injective tensor product and Haagerup tensor product.

References

1. Blecher, D. P. and Merdy, C. Le., Operator algebras and their modules-an operator space approach. London Mathematical Society Monographs, New series, vol. 30, The Clarendon Press, Oxford University Press, Oxford, 2004.
2. Effros, E. G. and Ruan, Z. J., Operator spaces, Claredon Press-Oxford, 2000.
3. Pisier, G., Introduction to operator space theory, Cambridge University Press, 2003.

RC(vii) - Advanced Operator Algebras

Unitary representations of locally compact groups: The involutive algebra L^1(G), representations of G and L^1(G), positive forms on L^1(G) and positive-definite functions, weak*-convergence and compact convergence of continuous positive-definite functions, pure positive-definite functions, square integrable positive definite functions, the C*-algebra of a locally compact group

Group C-algebras:* Group representations, amenability, free group, reduced C*-algebra of the free group

Tensor Products: Tensor products of Banach spaces, Tensor product of Hilbert spaces, Tensor products C*-algebras, Tensor products of W*-algebras

References

1. *C*-algebras by example, K. R. Davidson, American Mathematical Society*
2. *C*-algebras*, J. Dixmier, North Holland Publishing Company, 1977.
3. *Theory of Operator Algebras I*, M. Takesaki, Springer.
4. *Introduction to Tensor Product of Banach Spaces*, R. Ryan, Springer.
5. *Fundamentals of the Theory of Operator Algebras*, Volume II, R. V. Kadison and J. R. Ringrose, Academic Press.

RC(viii) - Symmetries and Differential Equations

Lie Groups of Transformations: Groups, Groups of Transformations, One-Parameter Lie Group of Transformations, Examples of One-Parameter Lie Groups of Transformations.

Infinitesimal Transformations: First Fundamental Theorem of Lie, Infinitesimal Generators, Invariant Functions, Canonical Coordinates, Examples of Sets of Canonical Coordinates, Invariant Surfaces, Invariant Curves, and Invariant Points.

Extended Transformations (Prolongations): Extended Group Transformations-One, Dependent and One Independent Variable, Extended Infinitesimal Transformations-One Dependent and One Independent Variable, Extended Transformations-One Dependent and n Independent Variables, Extended Infinitesimal Transformations-One, Dependent and n Independent Variables, Extended Transformations and Extended Infinitesimal Transformations-m Dependent and n Independent Variables.

Ordinary Differential Equations: Invariance of an Ordinary Differential Equation, First Order ODE's, Determining Equation for Infinitesimal Transformations of a First Order ODE, Determination of First Order ODE's Invariant Under a Given Group, Second and Higher Order ODE's, Reduction of Order by Differential Invariants, Examples of Reduction of Order, Determining Equations for Infinitesimal Transformations of an nth Order ODE, Determination of nth Order ODE's Invariant Under a Given Group, Applications to Boundary Value Problems for ODE’s.

Partial Differential Equations: Invariance of a Partial Differential Equation, Invariant Solutions, Mapping of Solutions to Other Solutions from Group Invariance of a PDE, Determining Equations for Infinitesimal Transformations of a k^{th} Order PDE, Invariance for Systems of PDE's, Determining Equations for Infinitesimal Transformations of a System of PDE's, Applications to Boundary Value Problems for PDE’s.

References:

(1). George W. Bluman, J. D. Cole, Similarity methods for differential equations, Springer New York (Verlag), 1974.
(2). George W. Bluman, Sukeyuki Kumei, Symmetries and Differential Equations, Springer New York, 1989.

RC(ix)-Chaotic Dynamical Systems

Theory and Application of Chaos in Dynamical systems, One dimensional map, Examples of Dynamical Systems, Stability of fixed points, Orbits, Graphical Analysis, Fixed and Periodic points, Quadratic family, Transition to Chaos.

Bifurcations of Chaotic Systems, Dynamics of a quadratic map, Saddle node Bifurcation, Period Doubling Bifurcation, Transcritical Bifurcation, Pitchfork Bifurcation.

Lyapunov Exponents, chaotic orbits, conjugacy and logistic map, Transition graphs and fixed points, Basin of attraction. Lorenz equations, strange attractors, Lorenz map, Simple properties of Lorenz equations, Chaos in Hamiltonian Systems, and Control and Synchronism of chaos.

Equilibria in Nonlinear Systems, Nonlinear Sinks and Sources, Saddles, Stability, Closed orbit and Limit Sets, Poincare map, Applications in physics, engineering and biology.

References:

(1). R. L. Deveny, A First Course in Chaotic Dynamical Systems, 2nd Edition, Westview Press.

(2). Steven H. Strogatz, Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Westview Press.

(3). Kathleen T. Alligood, Tim D. Sauer, James A. Yorke, Chaos: An Introduction to Dynamical Systems, Springer-Verlag, New York.

(4). L. Douglas Kiel and Euel W. Elliott, Chaos Theory in the Social Sciences, University of Michigan Press.

(5). M.W. Hirsch and S. Smale, Differential Equations, Dynamical Systems and an Introduction to Chaos, 3rd Edition, Academic Press, USA.

RC (x) - Recent Developments on Minimal Ring Extensions and APD's

Contents: Introduction to Minimal Ring extensions, Minimal Ring Homomorphisms, Overring, Finitely many intermediate rings property and related results, Finitely many subrings property and related results, Composites, Kaplansky Transform, Direct Products and ƛ-extensions, µ-extensions, P-extensions, i-domains. Results on minimal field extension.

Commutative Perfect Rings and almost Perfect Rings, Properties of Almost perfect domains, Valuation overrings of APDs, connection of APDs with other classes of domains, examples of APDs.

References:

1. Commutative Rings: New Research, John Lee, Nova Science Publication Inc., New York.
2. Multiplicative Ideal Theory in Commutative Algebra, J. W. Brewer, S. Glaz, W. J. Heinzer, B. M. Olberding, Springer, 2006
3. Commutative ring theory, H. Matsumura, Cambridge university press, 1989.
4. Basic commutative algebra, Balwant Singh, World scientific publishing co., 2011.

RC(xi)-Advanced commutative algebra

Direct limit, Inverse limit, Graded rings and modules, Associated graded rings, I-adic completion, Krull's intersection theorem, Hensel's lemma, Hilbert function, Hilbert polynomial, Dimension theory of Noetherian local rings, Regular local rings, UFD property of regular local rings, Hom functor, Tensor functor, I-torsion functor, Flat modules, Projective and injective modules, Complexes, Projective and injective resolution, Derived functor, Tor and ext functor, Minimal resolution, Regular sequences, Cohen-Macaulay rings and modules.

References:

1. H. Matsumura, Commutative ring theory, Cambridge university press, 1989.
2. Balwant Singh, Basic commutative algebra, World scientific publishing co., 2011.
3. D. Eisenbud, Commutative algebra with a view towards algebraic geometry, Springer verlag, 1995.
4. M.F. Atiyah & I.G. Macdonald, Introduction to commutative algebra, Addison Wesley, 1969.

RC(xii) - Banach Spaces of Analytic Functions

Analytic and Harmonic Functions in the unit disc: Cauchy and Poisson kernels, boundary values, Fatou's Theorem, H^p spaces.
The space H^1: The Helson Lowdenslager Approach, Szego's theorem, Dirichlet Algebras.
Factorization of H^p Functions: Inner and outer functions, Blaschke products and singular functions, Factorisation theorem. The Shift operator : The shift operator on H^2. Invariant subspaces for H^2 on the half plane, the shift on L^2 the vector valued case, representations on $H\infty$.

Text book: K. Hoffman, Banach Spaces of Analytic Functions, Dover Publications, 2007.

RC(xiii) - Banach Algebra Techniques in Operator Theory

Revision of Banach spaces and Geometry of Hilbert Space. Basic theory of Banach Algebras, The Disk Algebra. Multiplication operators and maximal abelian algebras. The Bilateral shift operator. C* algebras. The GelfandNaimark Theorem. Spectral Theorem. Functional Calculus. The Unilateral Shift Operator. Topelitz operators. The Spectrum of self-adjoint and analytic Toeplitz Operators.

R.G. Douglas, Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics 179, Springer, 1998

RC(xiv) - Conservation laws in Fluid Dynamics

Hyperbolic system of conservation laws, breakdown of smooth solution, genuine nonlinearity, weak solutions and jump condition, Riemann problem, entropy conditions, Convection, diffusion and heat transfer, two-phase flow, boundary layer flow, Free and Moving boundary problems.

Reference

1. Hyperbolic system of conservation laws and mathematical theory of shock waves, Peter D. Lax, SIAM, 1973
2. Quasilinear Hyperbolic Systems, Compressible Flows and Waves, V.D.Sharma, Chapman and Hall/ CRC, 2010
3. Free and Moving Boundary Problems, J. Crank, Oxford university press, New York, 1984
4. Boundary Layer Theory, H.Schlichting, K. Gersten, Springer, 2000
5. Thermo-Fluid Dynamics of Two-Phase Flow, M. Ishii, T.Hibiki, Springer, 2011

RC(xv) - Methods in Fluid Dynamics

Characteristics methods, Similarity methods, Self-similar solution and the method of Lie-group invariance, Perturbation methods, Homotopy perturbation methods, Homotopy analysis method, Adomian decomposition method, Variational method, Numerical method.

References

1. Similarity and Dimensional Method in Mechanics, L.I.Sedov, Mir Publisher, 1982
2. Symmetries and Differential Equation, G.W. Bluman and S. Kumei, Springer, 1989
3. Beyond Perturbation: Introduction to the Homotopy Analysis Method, S. Liao, Chapman and Hall/ CRC, 2004
4. Partial Differential Equation and Soliton Wave Theory, Abdul-Majid Waswas, Springer, 2009
5. Numerical Approximation of Hyperbolic System of Conservation Laws, E. Godlweski, P.A. Raviart, Springer, 1996
6. Fundamental of finite element method in heat and fluid flow, R.W.Lewis, P. Nithiarasu, K.N. Seetharamu, John-Wiley and Sons, 2004

RC(xvi) - Set-Valued Analysis

Order relations, Cone properties related to the topology and the order, Convexity notions for sets and set-valued maps, Solution concepts in vector optimization, Vector optimization problems with variable ordering structure, Solution concepts in set-valued optimization, Solution concepts based on vector approach, Solution concepts based on set approach, Solution concepts based on lattice structure, The embedding approach by Kuroiwa, Solution concepts with respect to abstract preference relations, Set-valued optimization problems with variable ordering structure, Approximate solutions of set-valued optimization problems, Relationships between solution concepts

Continuity notions for set-valued maps, Continuity properties of set-valued maps under convexity assumptions, Lipschitzproperties for single-valued and set-valued maps, Clarke's normal cone and subdifferential, Limiting cones and generalized differentiability, Approximate cones and generalized differentiability

References

1. Akhtar A. Khan, Christiane Tammer, Constantin Zălinescu, **Set-Valued Optimization: An Introduction with Applications**, Springer Verlag, 2015.
2. Regina S. Burachik and Alfredo N. Iusem, **Set-Valued Mappings and Enlargements of Monotone Operators**, Springer Verlag, 2008.
3. Guang-ya Chen, XuexiangHuang and Xiaogi Yang, **Vector Optimization: Set-valued and Variational Analysis**, Springer Verlag, 2005.

RC(xvii)- Scalarization in Multiobjective Optimization

Basics of multiobjectiveoptimization, Minimality notion, Polyhedral ordering cones, Pascoletti-Serafiniscalarization, Parameter set restriction for the Pascoletti-Serafiniscalarization, Modified Pascoletti-Serafiniscalarization, ε-constraint problem, Normal boundary intersection problem, Modified Polakproblem, Weighted Chebyshevnorm problem, Problem ofGourion and Luc, Generalized weighted sum problem, Weighted sum problem, Problem of Kaliszewski

Sensitivity results in partially ordered spaces, Sensitivity results in naturally ordered spaces, Sensitivity results for the ε-constraint problem

Adaptive parameter control, Quality criteria for approximations, Adaptive parameter control in the bicriteriacase, Algorithm for the Pascoletti-Serafiniscalarization, Algorithm for the ε-constraint scalarization, Algorithm for the normal boundary intersection scalarization, Algorithm for the modified Polakscalarization, Adaptive parameter control in the multicriteriacase

References

1. Gabriele Eichfelder, **Adaptive Scalarization Methods in Multiobjective Optimization**, Springer Verlag, 2008.
2. JohannesJahn, **Vector Optimization Theory, Applications, and Extensions**, Springer Verlag, 2011.
3. Kayan Deb, **Multi-Objective Optimization using Evolutionary Algorithms,** John Wiley & Sons, Chichester, 2001.

RC(xviii) - Fixed Point Theorems

Contractions, Banach Contraction Principle, Theorem of Edelstein, Picard–Lindelof Theorem.

Non expansive Maps, Schauder’sTheorem for non–expansive maps, Continuation Methods for Contractive and non–expansive mappings.

Some Applications of The Banach Contraction Principle, Some Extensions of Banach Contraction Principle for Single – Valued Mappings, Generalized distances, Some Extensions of Banach Contraction Principle under Generalized Distances, Multivalued versions of Banach Contraction Principle.

References :

[1] S. Almezel, Q. H. Ansari and M. A. Khamsi; Topics in Fixed Point Theory, Springer 2014.

[2] R. P. Agarwal, M. Meehan, D. O’ Regan; Fixed Point Theory And Applications, Cambridge University Press 2004.

RC(xix) - Applications of Fixed Point Theorems in Economics and Game Theory

Sperner's Lemma, The Knaster – Kuratowski –Mazurkiewicz Lemma, Brouwer's Fixed Point Theorem, The Fan – Browder Theorem, Kakutani's Theorem.

The maximum Theorem, Set with convex sections and a minimax Theorem, Variational inequalities, Price equilibrium and complementarity, Equilibrium of excess demand correspondences, Nash equilibrium of games and abstract economics, Walrasian equilibrium of an economy.

Reference :

[1] K.C.Border; Fixed Point Theorems with Applications to Economics and Game Theory; Cambridge University Press 1985.

RC (xx) - Mathematical- Biological Modeling

Continuous and Discrete population models for single species, Models for interacting populations, Dynamics of infectious diseases, Reaction Diffusion, Chemotaxis, Spatial pattern formation with reaction diffusion systems, Animal coat patterns and other practical applications of reaction diffusion mechanisms.

References:

1. J.D. Murray, Mathematical Biology I: An Introduction, Third Edition, Springer, 2002.
2. J.D. Murray, Mathematical Biology II: Spatial Models and Biomedical Applications, Third Edition, Springer, 2002.

RC (xxi) - Parallel Iterative Methods for Partial Differential Equations

Speedup; efficiency; Amdahl's law; point and block parallel relaxation algorithms (Jacobi, Gauss-Seidel, SOR); triangular matrix decomposition; quadrant interlocking factorisation method; red-black ordering; application to elliptic BVPs; parallel ADI algorithms; parallel multi-grid and domain decomposition method.

The alternating group explicit (AGE) method for two point BVPs (natural, derivative, mixed, periodic) and their convergence analysis; the modified AGE and smart AGE methods; the computational complexity of the AGE method; the Newton-AGE method.

Parabolic equation: AGE algorithm for diffusion-convection equation and its convergence analysis; stability analysis of more general scheme; coupled reduced AGE method; AGE method for fourth order parabolic equation.

Hyperbolic equation: Group explicit method for first and second order hyperbolic equations; stability analysis of Group Explicit method; AGE iterative method for first and second order hyperbolic equations.

Elliptic equation: Douglas-Rachford algorithm; BLAGE iterative algorithm with different boundary conditions; parallel implementation.

Books recommended:

1. Y. Saad, *Iterative Methods for Sparse Linear Systems*, SIAM, Philadelphia (2003).
2. L.A. Hageman and D.M. Young, *Applied Iterative Methods*, Dover publication, New York (2004).
3. Jianping Zhu, *Solving Partial Differential Equations on Parallel Computers*, World Scientific, New Jersey (1994).
4. D.J. Evans, *Group Explicit Methods for the Numerical Solution of Partial Differential Equations*, Gordon and Breach Science publisher, Amsterdam (1997).

RC(xxii) - Hyperspaces

The general notion of a Hyperspace, Topological invariance, Specified Hyperspaces. Convergence in hyperspaces, L-convergence, T_v -convergence, relation between L-convergence and T_v -convergence,

REFERENCES:

[1] Alejandro Wanes and Sam B. Nadler, Jr. Hyperspaces: Fundamentals and Recent Advances, Marcel Dekker, Inc. New York.

RC(xxiii) - Uniform Spaces

Uniformities and the uniform topology, Uniform continuity, Product uniformities, Completeness.

REFERENCES:

[1] John L. Kelly, General Topology, Springer-Verlag New York Berlin Heideberg.

[2] Stephen Willard, General Topology, Dover Publication, Inc, Mineola, New York.

RC(xxiv) – Introduction to Transformation Groups

Definition and fundamental properties oftopological Groups, Examples oftopological groups, subgroups,Isotropy groups,Isomorphism,Semi-direct products and Direct products, the Classical groups, Characteristic functions on compact groups.

Transformation groups and its fundamental properties, Examples of transformation groups, Group actions, Fixed point sets, Orbits and orbit spaces. Homogeneous spaces and equivariant maps, Induced transformation groups.

REFERENCES:

1. P.J. Hissins, Introduction to Topological Groups, L M S (Lecture Notes Series), Cambridge University Press, 1975.
2. M. L. Curtis, Matrix Groups, Springer, 1984.
3. G. E. Bredon, Introduction to Compact transformation Groups, Academic Press, 1972.
4. T. B. Singh, Elements of Topology, CRC Press (Taylor and Francis Group), 2013.

Syllabus for Ph.D Programme

MATH16-01	Research Methodology
MATH16-02	Distribution Theory and Calculus on Banach Spaces
MATH16-03	Operator Theory and Function Spaces
MATH16-04	Geometric Function Theory
MATH16-05	Introduction to Operator Algebras
MATH16-06	Advanced Frame Theory
MATH16-07	Rings and Modules
MATH16-08	Group Rings
MATH16-09	Advanced Commutative Algebra
MATH16-10	Differential Manifolds
MATH16-11	Topological Structure
MATH16-12	Chaos Theory
MATH16-13	Ergodic Theory
MATH16-14	Singular Homology Theory
MATH16-15	Convex and Nonsmooth Analysis
MATH16-16	Multi-objective Optimization
MATH16-17	Parallel Iterative Methods for Partial Differential Equations
MATH16-18	Advanced Compressible Flow
MATH16-19	Elliptic Curves and Cryptography
MATH16-20	Lie Group and Lie Algebras
MATH16-21	Representation of Nilpotent Lie Group
MATH16-22	Univalent Functions
MATH16-23	Theory of Differential Subordination
MATH16-24	Harmonic Mappings in the Plane
MATH16-25	Operator Spaces
MATH16-26	Symmetries and Differential Equations
MATH16-27	Chaotic Dynamical Systems

MATH16-28	Minimal Ring Extensions and APD’s
MATH16-29	Banach Spaces of Analytic Functions
MATH16-30	Banach Algebra Techniques in Operator Theory
MATH16-31	Conservation laws and Fluid Dynamics
MATH16-32	Methods in Fluid Dynamics
MATH16-33	Set-Valued Analysis
MATH16-34	Fixed Point Theorems in Non-Linear Analysis
MATH16-35	Applications of Fixed Point Theorems in Economics and Game Theory
MATH16-36	Introduction to Transformation Group
MATH16-37	Numerics of Partial Differential Equations
MATH16-38	Finite Difference Schemes for *K*-System Conservation Laws
MATH16-39	Computational Heat and Mass Transfer
MATH16-40	Uniform and Proximity structures on Topological Spaces
MATH16-41	Hyperspaces and Function spaces
MATH16-42	Introduction to Greedy Approximations

MATH16-01 : Research Methodology

Scientific research and literature survey. History of mathematics, finding and solving research problems, role of a supervisor, survey of a research topic, publishing a paper, reviewing a paper, research grant proposal writing, copyright issues, ethics and plagiarism.

Research tools. Searching google (query modifiers), MathSciNet, ZMATH, Scopus, ISI Web of Science, Impact factor, h-index, Google Scholar, ORCID, JStor, Online and open access journals, Virtual library of various countries

Scientific writing and presentation. Writing a research paper, survey article, thesis writing; LaTeX, PSTricks, Beamer, HTML and MathJaX

Software for Mathematics. Mathematica/Matlab/Scilab/GAP

Reference:

[1] J. Stillwell, Mathematics and its History, Springer International Edition, 4th Indian Reprint, 2005

[2] L. Lamport, LaTeX, a Document Preparation System, 2nd ed, Addison-Wesley, 1994.

[3] Norman E. Steenrod, Paul R. Halmos, Menahem M. Schiffer, Jean A. Dieudonne, How to Write Mathematics, American Mathematical Society, 1973.

[4] Nicholas J. Higham, Handbook of Writing for the Mathematical Sciences, Second Edition, SIAM, 1998.

[5] Donald E. Knuth, Tracy L. Larrabee, and Paul M. Roberts, Mathematical Writing, Mathematical Association of America Washington, D.C., 1989.

[6] Frank Mittelbach, Michel Goossens, Johannes Braams, David Carlisle, Chris Rowley, The LaTeX Companion, 2nd edition (TTCT series), Addison-Wesley, 2004.

[7] Michel Goossens, Frank Mittelbach, Sebastian Rahtz, Denis Roegel, Herbert Voss, The LaTeX Graphics Companion, 2nd edition (TTCT series), Addison-Wesley, 2004

[8] Mathtools documentation (http://mirrors.ctan.org/macros/latex/contrib/mathtools/mathtools.pdf)

[9] Pstricks documentation (http://tug.org/PSTricks/main.cgi?file=doc/docs)

[10] MathJax documentation (http://tug.org/PSTricks/main.cgi?file=doc/docs)

MATH16-02 : Distribution Theory and Calculus on Banach Spaces

Test functions and distributions, some operations with distributions, local properties of distributions, convolutions of distributions, tempered distributions and Fourier transform, fundamental solutions.

The Frechet derivative, chain rule and mean value theorems, implicit function theorem, extremum problems and Lagrange multipliers.

References.

[1] W. Cheney : Analysis for Applied Mathematics; Springer -Verlag, 2001.
[2] S. Kesavan : Topics in Functional Analysis and Applications; New Age International Publishers, 2008
[3] W. Rudin : Functional Analysis; Tata Mc-Graw Hill, 1991.
[4] Robert S. Strichartz : A guide to distribution theory and Fourier transforms; World Scientific Publishing Co., 2003.

MATH16-03 : Operator Theory and Function Spaces

Fredholm operators; semi-Fredholm operators; index of a Fredholm (semi- Fredholm) Operator; essential spectrum; Weyl spectrum and Weyl theorem; direct sums of operators, their spectra and numerical ranges; weighted shifts, their norms and spectral radii; normaloid, convexoid and spectraloid operators.

Invariant subspace problem; transitive, reductive and reflexive algebras; von-Neumann algebras.

Hardy spaces: Poisson's kernel; Fatou's theorem; zero sets of functions; multiplication, composition, Toeplitz and Hankel operators.

References.

[1] Vladimir V.Peller, Hankel operators and their applications, Springer, 2002.
[2] Nikolai L.Vasilevski, Commutative algebras of Toeplitz operators on Bergman space, Birkhauser, 2008.
[3] N.Young, An introduction to Hilbert space, Cambridge University Press, 1988.
[4] P.R.Halmos, A Hilbert space problem book, II Ed., D.VanNostrand Company, 1982.
[5] H.Radjavi and P.Rosenthal, Invariant subspaces, Springer Verlag, 1973.

MATH16-04 : Geometric Function Theory

Area theorem, growth, distortion theorems, coefficient estimates for univalent functions special classes of univalent functions. Lowner's theory and its applications; outline of de Banges proof of Bieberbach conjecture. Generalization of the area theorem, Grunsky inequalities, exponentiation of the Grunsky inequalities, Logarithmic coefficients. Subordination and Sharpened form of Schwarz Lemma

References.

[1] P. Duren, Univalent Functions, Springer, New York, 1983

[2] A. W. Goodman, Univalent Functions I & II, Mariner, Florida, 1983

[3] Ch. Pommerenke, Univalent Functions, Van den Hoek and Ruprecht, Göttingen, 1975.

[4] M. Rosenblum, J. Rovnyak, Topics in Hardy Classes and Univalent Functions, Birkhauser Verlag, 1994

[5] D. J. Hallenbeck, T. H. MacGregor, Linear Problems and Convexity Techniques in Geometric Function Theory, Pitman Adv. Publ. Program, Boston-London-Melbourne,1984.

[6] I.Graham, G. Kohr, Geometric Function Theory in One and Higher Dimensions, Marcel Dekker, New York, 2003.

MATH16-04 : Introduction to Operator Algebras

Basic definitions and examples of Banach*-algebras, Spectrum of a Banach algebra element, L^1-algebras and Beurling algebras, Tensor products of Banach algebras, Multiplicative linear functional, The Gelfand representations, Fourier algebra, Functional calculus of in C*-algebras, Continuity and homomorphisms, Approximate identities in C*-algebras, Quotient algebras of C*-algebras, Representations and positive linear functional, Double Commutation Theorem, Enveloping von Neumann algebra of a C*-algebra, Tensor products of C*-algebras.

References.

[1] J.Diximier, C*-algebras, North-Holland Amersdem, 1977.

[2] R.V. Kadison and J.R.Ringrose, Fundamentals of the theory of operator algebras, Graduate studies in Mathematics, 15, AMS, Providence, 1997.

[3] E.Kaniuth, A course in commutative Banach algebras, Springer Verlag, 2008.

[4] M.Takesaki, Theory of Operator algebras, Springer Verlag, 2001.

MATH16-06 : Advanced Frame Theory

An overview on frames.B-Spline Symmetric B-Splines. Frames of translates. The canonical dual frame .Compactly supported generators.An application to sam-pling theory.Shift-Invariant Systems, Frame-properties of shift-invariant system, Representations of the frame operator. Gabor Frames in $L^2(R)$. Basic Gabor frame theory, Tight Gabor frames, The duals of a Gabor frame, Explicit construction of dual frame pairs, Popular Gabor conditions Representations of the Gabor frame operator and duality. Wavelet frames in $L^2(R)$.

References.

[1] O. Christensen, Frames and bases (An introductory course), Birkhauser, Boston (2008).

[2] I. Daubechies, Ten Lectures on wavelets, SIAM, Philadelphia (1992).

[3] R. Young, A introduction to non-harmonic Fourier series, Academic Press, New York (revised edition 2001).

MATH16-07: Rings and Modules

Essential and superfluous submodules, Decomposition of rings, Generating and cogenerating, Modules with composition series, Fitting Lemma, Indecomposable decompositions of modules, Projective modules and generators, Radicals of projective modules, Projective covers, Injective hulls, Cogenerators, Flat modules. Singular submodules, Localization and maximal quotient rings. Essential finite generation, Finite dimensionality, Uniform modules and Goldie rings. Regular rings, Strongly regular rings, Unit regular rings, Right π - regular rings. Baer rings,Rickart rings. Baer*rings, Rickart*rings.

References.

[1] A.F.Anderson and K.R.Fuller: Rings and categories of modules, Springer- Verlag,1991 (Relevant sections of Ch. 2,3,4,5).
[2] S.K.Berberian : Baer Vings,SpringerVerlag, New York ,1972 (Ch.1, sections 3, 4).
[3] K.R.Goodeari : Ring theory (Non singular rings and modules), Marcel Dekker,Inc. New York (Relevant sections of Ch. 1,2,3).
[4] K.R.Goodeari : Von Neumann regular rings,Pitman, London, 1979 (Ch. 1,3,4).
[5] T.Y.Lam: Lectures on Modules and rings, Springer Verlag, 1998(Ch. 3 ,section 7(d)).

MATH16-08: Group Rings

Twisted Group Rings, Tensor Products, Idempotents, Finite groups, Augmentation annihilators, Group algebra as injective modules, Linear identities. The Center, Finite conjugate groups, Chain conditions.

References.

[1] D. S. Passman The Algebraic structure of Group Rings, Dover Publications (Reprint edition). 2011.
[2] S. K. Sehgal, Topics in Group Rings, Marcel Dekker, New York, and Basel, 1978.
[3] I.B.S. Passi, Group Rings and their Augmentation Ideals Lecture Notes in Mathematics 715, Springer, New York, 1979.
[4] A. A. BOVDI, Group Rings Uzhgorod State University, 1978.
[5] D. S. Passman, Infinite Group Rings, Pure and Applied Math. 6, Marcel Dekkar, New York, 1971.
[6] P. Rihenboim, Rings and Modules, Interscience Tracts in Pure and Applied Mathematics, No.6, Interscience, New York, 1969.
[7] C.P. Milies and S.K. Sehgal, An Introduction to Group Rings, Kluwer Academic Publishers, Dordrecht, 2002.

MATH16-09: Advanced Commutative Algebra

Direct limit, Inverse limit, Graded rings and modules, Associated graded rings, I-adic completion, Krull's intersection theorem, Hensel's lemma, Hilbert function, Hilbert polynomial, Dimension theory of Noetherian local rings, Regular local rings, UFD property of regular local rings, Hom functor, Tensor functor, I-torsion functor, Flat modules, Projective and injective modules, Complexes, Projective and injective resolution, Derived functor, Tor and ext functor, Minimal resolution, Regular sequences, Cohen-Macaulay rings and modules.

References:

[1] H. Matsumura, Commutative ring theory, Cambridge university press, 1989.

[2] Balwant Singh, Basic commutative algebra, World scientific publishing co., 2011.

[3] D. Eisenbud, Commutative algebra with a view towards algebraic geometry, Springer verlag, 1995.

[4] M.F. Atiyah & I.G. Macdonald, Introduction to commutative algebra, Addison Wesley, 1969.

MATH16-10 : Differential Manifolds

The derivative, continuously differentiable functions, the inverse function theorem, the implicit function theorem. Topological manifolds, partitions of unity, imbeddings and immersions, manifolds with boundary, submanifolds. Tangent vectors and differentials, Sard's theorem and regular values, vector fields and flows, tangent bundles, smooth maps and their differentials. Smooth manifolds, smooth manifolds with boundary, smooth sub-manifolds, construction of smooth functions.

References.

[1] G.E. Bredon, Topology and Geometry, Springer-verlag, 1993.

[2] L. Conlon, Differentiable Manifolds, Second Edition, Birkhauser,2003.

[3] A. Kosinski, Differential Manifolds, Academic Press, 1992.

[4] M. Spivak, A Comprehensive Introduction to Differential Geometry, Vol. I; Publish or Perish, 1979.

MATH16-11 : Topological Structures

Dimension Theory: Definition and basic properties of the three dimension function inc. Inc. and dim, Characterization and subset theorems, equality of dim X and dim βX equality of Ind X and Ind βX.

Paracompactness: Paracompactness and full normality, presentation of paracompactness under mappings, Hanai-Moritastone theorem, products of paracompact spaces, countable paracompactness, strong paracompactness characterizations of strong paracompactness in regular spaces, products and subspaces of strongly paracompact spaces, pointwise paracompactness Arens Dugundji theorem, collectionwise normal spaces, Ding's example of a normal space which is not collectionwise normal.

Bitopological Spaces: Basic concepts, subspaces and products Separation and covering axioms.

References.

[1] R. Engelking: General Topology, Polish Scientific Publishers Warszawa, 2^{nd} Ed., 1977.
[2] K. Nagami: DimensionTheory, Academic Press, New York, 1970.
[3] W.J. Pervin: Foundations of General Topology, Academic Press Inc., New York, 1964.
[4] S. Willard: General Topology, Addison-Wesley Publishing Co. Inc., 1970.

MATH16-12 : Chaos Theory

Topological transitivity: Examples and properties, Topological mixing: Examples and Properties, Transitivity and limit sets for maps on I , Characterizing topological mixing in terms of topological transitivity for maps on I , Sensitive dependence on initial conditions, Devaney's definition of chaos, Logistic maps and shift maps as chaotic maps, Period three implies chaos, Relation between transitivity and chaos on I .

Topological Entropy: Definitions, Entropy of interval maps, Horseshoes, Entropy of cycles, Continuity properties of the Entropy, Entropy of shift spaces, Entropy for circle maps, Various other definitions of Chaos and their interrelationships.

References.

[1] L. Alseda, J. Llibre, M. Misiurewicz, Combinatorial Dynamics and Entropy inDimension One, Advanced Series in Nonlinear Dynamics, 2000.

[2] L. S. Block and W. A. Coppel, Dynamics in One dimension, Springer, 1992.

[3] R. L. Devaney, A First Course in Chaotic Dynamical Systems, Westview Press, 1992.

[4] D. Hanselman and B. Littlefiels, Mastering MATLAB, Pearson Education,2005.

[5] Clark Robinson, Dynamical Systems, Stability, Symbolic Dynamics and Chaos, CRC press, 1999.

[6] S. Ruette, Chaos for continuous interval maps: A survey of relationship between various sorts of chaos, 2003.

[7] Introduction to Dynamical Systems, Brin and Stuck, Cambridge Univ. Press, 2002

MATH16-13 : Ergodic Theory

Measure preserving transformations and examples, Recurrence, Poincare's Recurrence theorem, Ergodicity, ergodicity of shift transformations

Ergodic theorems of Birkhoff and Von Neuman, Mixing, Weak-mixing and their characterizations, the isomorphism problem: conjugacy, Spectral equivalence, Transformations with discrete spectrum, Entropy, Kolmogorov-Sinai theorem, K-systems examples of calculation of entropy, Unique ergodicity, uniformly distributed sequences, applications to Diophantine approximation.

References.

[1] P. R. Halmos, Lectures on Ergodic Theory, American Mathematical Society, 2006

[2] M. G. Nadkarni, Basic Ergodic Theory, Birkhauser Verlag, 1998.

[3] Peter Walters, An Introduction to Ergodic Theory, Springer.

MATH16-14 : Singular Homology Theory

Singular complex and homology groups, functorial properties, relative homology groups, the Eilenberg-Steenrod axioms of homology theory.Long exact sequences. The reduced homology groups, the Mayer-Vietoris sequence. Homology of spheres. The degree of self maps of n-sphere, The Brouwer's fixed point theorem, Hairy Ball Theorem, Lusternik-Schnirelmann Theorem, Jordan-Brouwer Separation Theorem, Invariance of Domain.

References:

[1] E H Spanier, Algebraic Topology, Springer Verlag, 1989.

[2] G E Bredon, Topology and geometry, Springer Verlag, 2005.

[3] A Dold, Lectures on Algebraic Topology, Springer-Verlag, Second Edition 1980.

[4] J JRotman, An Introduction to Algebraic Topology, Springer Verlag, 1988.

[5] M.J. Greenberg and J.R. Harper, Algebraic Topology- A first course, Addison-Wesley Publishing Company, Inc. 1981.

[6] W S Massey, A Basic Course in Algebraic topology, Springer- Verlag, 1991.

MATH16-15 : Convex and Nonsmooth Analysis

Convex sets, Convexity-preserving operations for a set, Relative Interior, Asymptotic cone, Separation theorems, Farkas Lemma, Conical approximations of convex sets, Bouligand tangent and normal cones. Convex functions of several variables, Affine functions, Functional operations preserving convexity of function, Infimal convolution, Convex hull and closed convex hull of a function, Continuity properties, Sublinear functions, Support function, Norms and their duals, Polarity. Subdifferential of convex functions, Geometric construction and interpretation, properties of subdifferential, Minimality conditions, Mean-value theorem, Calculus rules with subdifferentials, Subdifferential as a multifunction, monotonicity and continuity properties of the subdifferential, Subdifferential and limits of gradients.

References.

[1] Convex, Analysis and Minimization Algorithms I, Jean-Baptiste Hiriart-Urruty and Claude Lemarechal, Springer- Verlag, Berlin, 1996.

[2] Convex Analysis and Nonlinear Optimization : Theory and Examples, Jonathan M. Borwein Adrian and S. Lewis, CMS Books in Mathematics, Springer Verlag, New York, 2006.

[3] Convex Analysis, R. Tyrrell Rockafellar, Priceton University Press, Princeton, New Jersey, 1997.

MATH16-16 : Multi-objective Optimization

Multiple Objective Linear Programming Problem, Multiple Criteria Examples, Utility Functions,Non Dominated Criteria Vectors and Efficient Points, Point Estimate Weighted Sums Approach, Optimal Weighting Vectors, Scaling and Reduced Feasible Region Methods, Vector Maximum Algorithm. Formulation of the Multiple Objective Model, Method of Solutions, Augmented Goal Programming, Interactive Multiple Objective Methods. Multiple Objective Linear Fractional Programming. Multiple Objective Non linear Programming Problem, Efficiency and Non- Dominance, Weakly and Strictly Efficient Solutions, Proper Efficiency and Proper Non- Dominance. Weighted Sum Scalarization : (Weak) Efficiency, Proper Efficiency, Optimality Conditions. Scalarization Techniques: The €-Constraint Method, The Hybrid Method, The Elastic Constraint Method and Benson's Method.

<u>References:</u>

[1] Ralph E.Steuer : Multi Criteria Optimization: Theory, Computation, and Application, John Wiley and Sons, 1986. Chapters-1, 6, 7, 8, 9, 12.

[2] James P. Ignizio : Linear Programming in Single and Multiple Objective Systems, Prentice-Hall Inc. , Englewood Cliffs, N.J 1981. Chapters- 16, 17, 20.

[3] Matthias Ehrgott: Multicriteria Optimization, Springer, Berlin, Heidelberg, 2005, Second Edition, Chapters- 2, 3,4.

MATH16-17 : Parallel Iterative methods for Partial Differential Equations

Speedup; efficiency; Amdahl's law; point and block parallel relaxation algorithms (Jacobi, Gauss-Seidel, SOR); triangular matrix decomposition; quadrant interlocking factorisation method; red-black ordering; application to elliptic BVPs; parallel ADI algorithms; parallel conjugate-gradient method; parallel multi-grid method; parallel domain decomposition method.

The alternating group explicit method for two point BVPs (natural, derivative, mixed, periodic) and their convergence analysis; the MAGE and NAGE methods; the computational complexity of the AGE method; the Newton-AGE method.

Parabolic equation: AGE algorithm for diffusion-convection equation and its convergence analysis; stability analysis of more general scheme; CAGE method; AGE method for fourth order parabolic equation.

Hyperbolic equation: Group explicit method for first and second order hyperbolic equations; GER, GEL, GAGE, GEU, GEC algorithms; stability analysis of GE method; AGE iterative method for first and second order hyperbolic equations.

Elliptic equation: Douglas-Rachford algorithm; BLAGE iterative algorithm with different boundary conditions; AGE-DG algorithm; parallel implementation.

This course consists of theory paper and computer practical.

References.

[1] Y. Saad, Iterative Methods for Sparse Linear Systems, SIAM, Philadelphia (2003).
[2] L.A. Hageman and D.M. Young, Applied Iterative Methods, Dover publication, New York (2004).
[3] D.M. Young, Iterative Solution of Large Linear Systems, Academic Press, New York (1971).
[4] Jianping Zhu, Solving Partial Differential Equations on Parallel Computers, World Scientific, New Jersey (1994).
[5] D.J. Evans, Group Explicit Methods for the Numerical Solution of Partial Differential Equations, Gordon and Breach Science publisher, Amsterdam (1997).

MATH16-18 : Advanced Compressible Flows

One-dimensional gas flow (with perfect and van der Waals gas, gravitation, viscosity, heat addition, and conduction), Diffusion, Shock waves (discontinuity surface, jump condition, strength, thickness, reflection, structure, heat addition and MHD effects), Detonation and Deflagration waves, Methods of solution of compressible flow problems. Dimensional analysis and similarity method, Self-similar motion of spherical, cylindrical and plane waves in a gas. Two dimensional subsonic and supersonic flow with linearized theory, Two dimensional subsonic potential flows(Rayleigh-Janzen Method), Two dimensional supersonic flow with method of characteristics. Anisentropic rotational flow of inviscid compressible fluid.

References.

1. Similarity and Dimensional Method in Mechanics, L.I.Sedov, Mir Publisher, 1982
2. Fluid Mechanics, L.D.Landau and E.M.Lifshitz, Pragamon Press, 1989
3. Introduction to the theory of compressible flow, S.I.Pai, D. Van Nostrand Company, 1958
4. Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Ya. B. Zel'dovich, and Yu.P. Raizer, Academic Press, 1966.
5. Introductory Fluid Mechanics, J.Katz , Cambridge University Press, 2010

MATH16-19 : Elliptic Curves and Cryptography

Finite field arithmetic, Geometry and arithmetic of elliptic curves, torsion points, Elliptic curves over finite fields,Determination of number of points on elliptic curves,Discrete Logarithm Problem, Elliptic curve cryptography - including key agreement and key trans- port, Other applications such as factoring, primality testing, Elliptic curves over Q, Elliptic curves over C, complex multiplication, Divisors, Isogenies, Pairings and cryptography from pairings.

References:

[1] Lawrence C. Washington, Elliptic Curves, Number Theory and Cryptography, CRC Press, 2008.

[2] Darrel Hankerson, Alfred Menezes, Scott Vanstone, Guide to Elliptic curve Cryptog- raphy, Springer, 2004.

[3] Ian F. blake, Gadiel Seroussi, Nigel p. Smart, Advances in Elliptic curve cryptography, London University Press, 2005.

MATH16-20 : Lie Groups and Lie Algebras

Unit -I : Differential Manifolds Topological manifolds, Charts, Atlases and smooth structure, Smooth maps and diffeomorphism, Partitions of Unity, Tangent space, Tangent map, Vector fields and 1-forms.

Unit -II : Lie Groups Definition and examples, Linear Lie groups, Lie group homomorphism, Lie algebra and the exponential map, Adjoint representation, Homogeneous spaces, Baker-Campbell-Housdorff formula.

Unit -III : Lie Algebras Definition and examples, Classical Lie algebras, Solvable and nilpotent Lie algebras, Lie and Engel theorems, Semisimple and reductive algebras, Semisimplicity of Classical Lie algebras, Killing form and Cartan criterion, Cartansubalgebra, root decomposition and root systems, Weyl group and Weyl chambers, Dynkin diagrams, Classsification of simple Lie algebras.

Unit -IV : Partial Differential Equations on Manifolds Partial differential operators and formal adjoints, Sobolev spaces in R^n, Elliptic estimates in R^n, Elliptic regularity, Fredholm theory and spectral theory of Laplacian.

Suggested Texts:

1. S. Kumaresan, Differential Geometry and Lie Groups, Hindustan Book Agency.

2. Alexander KirillovJr, An Introduction to Lie Groups and Lie Algebras, Cambridge University Press.

3. James Humphreys, Introduction to Lie Algebras and Representation Theory, Springer.

4. Brian Hall, Lie Groups, Lie Lagebras, and Representations: An Elementary Introduction, Second Edition, Springer.

5. J. M. Lee, Manifolds and Differential Geometry, Graduate Studies in Mathematics vol 107, AMS.

6. Liviu I Nicolaescu, Lectures on Geometry of Manifolds, Second Edition, World Scientific.

7. A. C. Pipkin, A Course on Integral Equations, Text in Applied Mathematics Series, Springer.

MATH16-21 : Representation of Nilpotent Lie Group

Basic facts about Lie groups and Lie algebras, Nilpotent Lie groups, Coadjoint orbits and the dual of g, Some generalities on representations, Elements of kirillov theory, Proof of basic theorems, subgroups of condimension 1 and representations.

References

1. L.J. Corwin and F.D. Grenleaf, Representations of nilpotent Lie groups and their applications, Cambridge University Press, 1990
2. V.S. Varadarajan, Lie groups, Lie algebras and their representations, Prentice-Hall, 1974.
3.

MATH16-22 : Univalent Functions

Univalent functions, area theorems, Bieberbach theorem and itsapplications, subclasses of starlike and convex functions and theirgeneralizations, functions with positive real part, typically realfunctions.

Close-to-convex functions and the functions of bounded boundaryrotation, bounded functions, radius problems and Koebe domains,combination and convolutions of univalent functions, Integrals andintegral inequalities, meromorphic functions.

References.

[1]] A. W. Goodman, Univalent Functions I & II, Mariner, Florida, 1983.

[2] P. Duren, Univalent Functions, Springer, New York, 1983

[3] Ch. Pommerenke, Univalent Functions, Van den Hoek and Ruprecht,Göttingen, 1975.

MATH16-23 : Theory of Differential Subordination

Jack-Miller-Mocanu Lemma, Admissible functions and fundamentaltheorems, open door lemma and integral existence theorem, first orderdifferential subordination, Briot-Bouquet differential subordinations,and its generalizations and applications, integral operator, subordination preserving integral operators .

Second order differential subordinations, integral operatorspreserving functions with positive real part, bounded functions,averaging operators, Hypergemetric functions, Schwarzian derivative,applications to starlikeness and convexity.

References

[1] S. S. Miller and P. T. Mocanu, Differential Subordinations. Theoryand. Applications, Marcel Dekker Inc., New York, Basel, 2000.

[2] T. Bulboacˇa, Differential Subordinations and Superordination:Recent Results, Cluj-Napoca,2005.

MATH16-24 : Harmonic Mappings in the Plane

Harmonic mappings, Argument principle, Dirichlet problem, criticalpoints of harmonic mappings, Lewy's theorem, Heinz's theorem, Rado'stheorem

The Rado-Kneser-Choquet theorem, Shear construction, structure ofconvex mappings, covering theorems and coefficient bounds

Harmonic self mappings of the disk, normalization and normality ofharmonic univalent functions, Harmonic Koebe functions andcoefficient conjectures, extremal problems, typically real andstarlike functions, problems and conjectures in planar harmonicmappings.

Text:

[1] P.L. Duren, Harmonic Mappings in the Plane, Cambridge Univ.Press, Cambridge, 2004.

[2] D. Bshouty and A. Lyzzaik, Problems and Conjectures in PlanarHarmonic Mappings, J. Analysis, Volume 18 (2010), 69–81.

MATH16-25 : Operator Spaces

Operator Spaces (concrete and abstract), Completely bounded maps, subspaces, quotients, products, Dual spaces, conjugates, mapping spaces, opposite, representation theorem, The min and max quantizations, Arveson-Wittstock theorem, Column and Row Hilbert spaces. Projective tensor product, injective tensor product and Haagerup tensor product.

References

1. Blecher, D. P. and Merdy, C. Le., Operator algebras and their modules-an operator space approach. London Mathematical Society Monographs, New series, vol. 30, The Clarendon Press, Oxford University Press, Oxford, 2004.
2. Effros, E. G. and Ruan, Z. J., Operator spaces, Claredon Press-Oxford, 2000.
3. Pisier, G., Introduction to operator space theory, Cambridge University Press, 2003.

MATH16-26 : Symmetries and Differential Equations

Lie Groups of Transformations: Groups, Groups of Transformations, One-Parameter Lie Group of Transformations, Examples of One-Parameter Lie Groups of Transformations.

Infinitesimal Transformations: First Fundamental Theorem of Lie, Infinitesimal Generators, Invariant Functions, Canonical Coordinates, Examples of Sets of Canonical Coordinates, Invariant Surfaces, Invariant Curves, and Invariant Points.

Extended Transformations (Prolongations): Extended Group Transformations-One, Dependent and One Independent Variable, Extended Infinitesimal Transformations-One Dependent and One Independent Variable, Extended Transformations-One Dependent and n Independent Variables, Extended Infinitesimal Transformations-One, Dependent and n Independent Variables, Extended Transformations and Extended Infinitesimal Transformations-m Dependent and n Independent Variables.

Ordinary Differential Equations: Invariance of an Ordinary Differential Equation, First Order ODE's, Determining Equation for Infinitesimal Transformations of a First Order ODE, Determination of First Order ODE's Invariant Under a Given Group, Second and Higher Order ODE's, Reduction of Order by Differential

Invariants, Examples of Reduction of Order, Determining Equations for Infinitesimal Transformations of an nth Order ODE, Determination of nth Order ODE's Invariant Under a Given Group, Applications to Boundary Value Problems for ODE's.

Partial Differential Equations: Invariance of a Partial Differential Equation, Invariant Solutions, Mapping of Solutions to Other Solutions from Group Invariance of a PDE, Determining Equations for Infinitesimal Transformations of a k^{th} Order PDE, Invariance for Systems of PDE's, Determining Equations for Infinitesimal Transformations of a System of PDE's, Applications to Boundary Value Problems for PDE's.

References:

(1). George W. Bluman, J. D. Cole, Similarity methods for differential equations, Springer New York (Verlag), 1974.
(2). George W. Bluman, SukeyukiKumei, Symmetries and Differential Equations, Springer New York, 1989.

MATH16-27 : Chaotic Dynamical Systems

Theory and Application of Chaos in Dynamical systems, One dimensional map, Examples of Dynamical Systems, Stability of fixed points, Orbits, Graphical Analysis, Fixed and Periodic points, Quadratic family, Transition to Chaos.

Bifurcations of Chaotic Systems, Dynamics of a quadratic map, Saddle node Bifurcation, Period Doubling Bifurcation, Transcritical Bifurcation, Pitchfork Bifurcation.

Lyapunov Exponents, chaotic orbits, conjugacy and logistic map, Transition graphs and fixed points, Basin of attraction. Lorenz equations, strange attractors, Lorenz map, Simple properties of Lorenz equations, Chaos in Hamiltonian Systems, and Control and Synchronism of chaos.

Equilibria in Nonlinear Systems, Nonlinear Sinks and Sources, Saddles, Stability, Closed orbit and Limit Sets, Poincare map, Applications in physics, engineering and biology.

References:

(1). R. L. Deveny, A First Course in Chaotic Dynamical Systems, 2nd Edition, Westview Press.
(2). Steven H. Strogatz, Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Westview Press.
(3). Kathleen T. Alligood, Tim D. Sauer, James A. Yorke, Chaos: An Introduction to Dynamical Systems, Springer-Verlag, New York.
(4). L. Douglas Kiel and Euel W. Elliott, Chaos Theory in the Social Sciences, University of Michigan Press.
(5). M.W. Hirsch and S. Smale, Differential Equations, Dynamical Systems and an Introduction to Chaos, 3rd Edition, Academic Press, USA.

MATH16-28 : Minimal Ring Extensions and APD's

Contents: Introduction to Minimal Ring extensions, Minimal Ring Homomorphisms, Overring, Finitely many intermediate rings property and related results, Finitely many subrings property and related results, Composites, Kaplansky Transform, Direct Products and ƛ-extensions, μ-extensions, P-extensions, i-domains. Results on minimal field extension.

Commutative Perfect Rings and almost Perfect Rings, Properties of Almost perfect domains, Valuation overrings of APDs, connection of APDs with other classes of domains, examples of APDs.

References:

1. Commutative Rings: New Research, John Lee, Nova Science Publication Inc., New York.
2. Multiplicative Ideal Theory in Commutative Algebra, J. W. Brewer, S. Glaz, W. J. Heinzer, B. M. Olberding, Springer, 2006
3. Commutative ring theory, H. Matsumura, Cambridge university press, 1989.
4. Basic commutative algebra, Balwant Singh, World scientific publishing co., 2011.

MATH16-29 : Banach Spaces of Analytic Functions

Analytic and Harmonic Functions in the unit disc: Cauchy and Poisson kernels, boundary values, Fatou's Theorem, H^pspaces.
The space H^1: The HelsonLowdenslager Approach, Szego's theorem, Dirichlet Algebras.
Factorization of H^pFunctions: Inner and outer functions, Blaschke products and singular functions, Factorisation theorem. The Shift operator : The shift operator on H^2.Invariant subspaces for H^2 on the half plane, the shift on L^2 the vector valued case, representations on H^∞.

Text book: K. Hoffman, Banach Spaces of Analytic Functions, Dover Publications, 2007.

MATH16-30 : Banach Algebra Techniques in Operator Theory

Revision of Banach spaces and Geometry of Hilbert Space.Basic theory of Banach Algebras, The Disk Algebra.Multiplication operators and maximal abelian algebras. The Bilateral shift operator. C* algebras.The GelfandNaimark Theorem.Spectral Theorem.Functional Calculus.The Unilateral Shift Operator.Topelitz operators.The Spectrum of self-adjoint and analytic Toeplitz Operators.

R.G. Douglas, Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics 179, Springer, 1998

MATH16-31 : Conservation laws in Fluid Dynamics

Hyperbolic system of conservation laws, breakdown of smooth solution, genuine nonlinearity, weak solutions and jump condition, Riemann problem, entropy conditions, Convection, diffusion and heat transfer, two-phase flow, boundary layer flow, Free and Moving boundary problems.

Reference

1. Hyperbolic system of conservation laws and mathematical theory of shock waves, Peter D. Lax, SIAM, 1973
2. Quasilinear Hyperbolic Systems, Compressible Flows and Waves, V.D.Sharma, Chapman and Hall/ CRC, 2010
3. Free and Moving Boundary Problems, J. Crank, Oxford university press, New York, 1984
4. Boundary Layer Theory, H.Schlichting, K. Gersten, Springer, 2000
5. Thermo-Fluid Dynamics of Two-Phase Flow, M. Ishii, T.Hibiki, Springer, 2011

MATH16-32 : Methods in Fluid Dynamics

Characteristics methods, Similarity methods, Self-similar solution and the method of Lie-group invariance, Perturbation methods, Homotopy perturbation methods, Homotopy analysis method, Adomian decomposition method, Variational method, Numerical method.

References

6. Similarity and Dimensional Method in Mechanics, L.I.Sedov, Mir Publisher, 1982
7. Symmetries and Differential Equation, G.W. Bluman and S. Kumei, Springer, 1989
8. Beyond Perturbation: Introduction to the Homotopy Analysis Method, S. Liao, Chapman and Hall/ CRC, 2004
9. Partial Differential Equation and Soliton Wave Theory, Abdul-MajidWaswas, Springer, 2009
10. Numerical Approximation of Hyperbolic System of Conservation Laws, E. Godlweski, P.A. Raviart, Springer, 1996
11. Fundamental of finite element method in heat and fluid flow, R.W.Lewis, P. Nithiarasu, K.N. Seetharamu, John-Wiley and Sons, 2004

MATH16-33 : Set-Valued Analysis

Order relations, Cone properties related to the topology and the order, Convexity notions for sets and set-valued maps, Solution concepts in vector optimization, Vector optimization problems with variable ordering structure, Solution concepts in set-valued optimization, Solution concepts based on vector approach, Solution concepts based on set approach, Solution concepts based on lattice structure, The embedding approach by Kuroiwa, Solution concepts with respect to abstract preference relations, Set-valued optimization problems with variable ordering structure, Approximate solutions of set-valued optimization problems, Relationships between solution concepts

Continuity notions for set-valued maps, Continuity properties of set-valued maps under convexity assumptions, Lipschitzproperties for single-valued and set-valued maps, Clarke's normal cone and subdifferential, Limiting cones and generalized differentiability, Approximate cones and generalized differentiability

References

1. Akhtar A. Khan, Christiane Tammer, Constantin Zălinescu, Set-Valued Optimization: An Introduction with Applications, Springer Verlag, 2015.
2. Regina S. Burachik and Alfredo N. Iusem, Set-Valued Mappings and Enlargements of Monotone Operators, Springer Verlag, 2008.
3. Guang-ya Chen, XuexiangHuang and Xiaogi Yang, Vector Optimization: Set-valued and Variational Analysis, Springer Verlag, 2005.

MATH16-34 : Fixed Point Theorems in Non-Linear Analysis

Contractions, Banach Contraction Principle, Theorem of Edelstein, Picard–Lindelof Theorem.Non expansive Maps, Schauder'sTheorem for non–expansive maps, Continuation Methods for Contractive and non–expansive mappings.Some Applications of The Banach Contraction Principle, Some Extensions of Banach Contraction Principle for Single – Valued Mappings, Generalized distances, Some Extensions of Banach Contraction Principle under Generalized Distances, Multivalued versions of Banach Contraction Principle.

References :

[1] S. Almezel, Q. H. Ansari and M. A. Khamsi; Topics in Fixed Point Theory, Springer 2014.

[2] R. P. Agarwal, M. Meehan, D. O' Regan; Fixed Point Theory And Applications, Cambridge University Press 2004.

MATH16-35 : Applications of Fixed Point Theorems in Economics and Game Theory

Sperner's Lemma, TheKnaster – Kuratowski –Mazurkiewicz Lemma, Brouwer's Fixed Point Theorem, The Fan – Browder Theorem, Kakutani's Theorem. The maximum Theorem, Set with convex sections and a minimax Theorem, Variational inequalities, Price equilibrium and complementarity, Equilibrium of excess demand correspondences, Nash equilibrium of games and abstract economics, Walrasian equilibrium of an economy.

Reference :

[1] K.C.Border; Fixed Point Theorems with Applications to Economics and Game Theory; Cambridge University Press 1985.

MATH16-36 : Introduction to Transformation Groups

Definition and fundamental properties oftopological Groups, Examples oftopological groups, subgroups,Isotropygroups,Isomorphism,Semi-direct products and Direct products, the Classical groups, Characteristic functions on compact groups.

Transformation groups and its fundamental properties, Examples of transformation groups, Group actions, Fixed point sets, Orbits and orbit spaces. Homogeneous spaces and equivariant maps, Induced transformation groups.

REFERENCES:

1. P.J. Hissins, Introduction to Topological Groups, L M S (Lecture Notes Series), Cambridge University Press, 1975.
2. M. L. Curtis, Matrix Groups, Springer, 1984.
3. G. E. Bredon, Introduction to Compact transformation Groups, Academic Press, 1972.
4. T. B. Singh, Elements of Topology, CRC Press (Taylor and Francis Group), 2013.

MATH16-37 : Numerics of Partial Differential Equations

Finite Difference Methods for Parabolic, Hyperbolic and Elliptic PDEs of two and Three dimensions and their Consistency, Stability and Convergence, Dispersion and Dissipation analysis of PDEs and its Finite Difference Schemes, Artificial Dissipation, More Dissipation, Discontinuous Solutions, Finite Difference Schemes for the systems of Parabolic and Hyperbolic PDEs, Courant FriedrichsLewy condition for systems. Analysis of well-posed initial value problem of Parabolic, Hyperbolic systems, Kreiss matrix Theorem, Convergence estimates for Parabolic and Hyperbolic PDEs.

References

[1] John C. Strikwerda, *Finite Difference Schemes and Partial Differential equations*, SIAM, Philadelphia (2004).

[2] J.W. Thomas, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer-Verlag New York (1995).

[3]Quarteroni, A and Valli, A. *Numerical Approximation of Partial Differential Equations*, Springer, (1997).

[4]Ueberrhuber, C. W. , *Numerical Computation: Methods, Software and Analysis*, Springer, (1997).

[5]Axelsson, O*. Iterative Solution Methods,* Cambridge University Press,
(1994).

MATH16-38 : Finite Difference Schemes for *K*-System Conservation Laws

Theory of Scalar and *K* - System of Conservation Laws, Finite Difference Schemes for Conservation Laws. Difference Schemes for Scalar Conservation Laws: Godunov Scheme, TVD Scheme, Flux-Limiter Methods, Slope-Limiter Methods, Modified Flux Method. Finite Difference Schemes for *K*-System Conservation Laws, High Resolution Schemes for Linear *K*-System conservation Laws, Flux-Limiter Schemes for Linear *K*-System Conservation Laws, Slope Limiter Schemes for Linear *K*-System Conservation Laws, A Modified Flux Scheme for Linear *K*-System Conservation Laws, Approximate Reimann solvers, Difference Schemes for two dimensional Conservation Laws.

References

[1] J.W. Thomas, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer-Verlag New York (1995).

[2] Kroner, D, *Numerical Schemes for Conservation Laws*, John Wiley (1997).

[3]LeVeque, R.J, *Numerical Methods for Conservation Laws*, Birkhauser (1992).

[4]LeVeque, R. J, *Finite Volume methods for Hyperbolic Problems*, Cam-

bridge University Press (2002).

[5] Godlewski and Raviart, P, *Numerical Approximation of Hyperbolic Systems of Conservation Laws*, Springer(1995).

[6] John C. Strikwerda, *Finite Difference Schemes and Partial Differential equations*, SIAM, Philadelphia (2004).

MATH16-39 : Computational Heat and Mass Transfer

Introduction and basics of heat and mass transfer, Modes of heat transfer, Fourier's law, Conductivity, Diffusivity, Analogy between heat and mass transfer, Mass diffusion, Fick's Law, Transient mass diffusion, Steady and transient heat conduction, 1-D and 2-D Heat conduction, General heat conduction equation, Boundary and initial conditions, Heat generation, Introduction to convection: Fundamentals, Velocity and thermal boundary layer, Laminar, Turbulent flows, Conservation equations for mass, momentum and energy, Solution of boundary layer equations, Analogy between heat and momentum transfer, Non-dimensional numbers, Numerical methods: Solution of heat and mass transfer equations using finite difference and finite volume methods, Different explicit and implicit methods of finite differences, Different finite volume schemes for steady and transient Convection-Diffusion equations, Methods for solving finite difference and finite volume discretization equations, Consistency, stability and convergence of finite difference methods.

References

[1]F.P.Incropera and D.P.Dewitt, *Fundamental of Heat and Mass Transfer*, Wiley, USA, 1990.

[2]Yunus A. Cengel, *Heat Transfer*, McGra-Hill, NewYork.USA, 2004.

[3] J.P.Holmann, *Heat Transfer*, McGra-Hill, NewYork. USA, 2009.

[4] S.V. Patanker, *Numerical Heat Transfer and Fluid Flow*, Taylor and Francis, Hemisphere Pub.Comp., USA, 2004.

[5]H.K.Versteeg and W. Malalasekhera, *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*, Pearson, 2007

MATH16-40 : Uniform and Proximity structures on Topological Spaces

Uniform Spaces [1]:Denition of Uniform spaces, Discrete uniformity, Trivial uniformity, Base for a uniformity, Separated Uniformities, Totally bounded Uniformities, Uniform Continuities, Product uniformities, Induced Uniformities. 15 L

Uniform Topology [1]: Uniform neighbourhood, Closure in Uniform spaces, Uniformization of Compact Hausdor spaces, Cauchy Sequence, Cauchy Filters. 5L

Proximity Space [3]:Kuratowski closure axioms, Cech Closure axioms, Definition of proximity, Basic proximity, L-proximity, separated proximity, weakly regular spaces, compatible topology,s-axioms, Fine L-proximity, subspace proximity, Examples of proximity, Functionally separating proximity, Uniformity induced-proximity, proximal continuity, Clusters, Clans and Bunches, Wallman topology, Bases and subbases for a proximity. 20L

References

[1] I.M.James,Topological and uniform spaces,Springer-Verlag, New York Berlin Heidelberg,(1987).

[2] SomashekharNaimpally, Proximity Approach to Problems in Topology and Analysis, OldenbourgVerlag, Munich,Germany, (2009).

Suggested Readings

[1] John L. Kelly, General Topology, Springer-Verlag, New York Berlin Heideberg (1957).

[2] Stephen Willard, General Topology, Dover Publication, Inc, Mineola, New York (1970).

MATH16-41 : Hyperspaces and Function spaces

Hyperspace Topologies :

Topology for Hyperspaces [1]:The general notion of a Hyperspace, Vietoris topology for CL(X), Base for a Hyperspace topology, Topological Invariance, Specified Hyperspaces, The Hausdorff Metric. 10 L

Hyperspace Topologies [2]:Hit-and-miss topology, Far set, upper-far topology, Vietoris and Proximal topologies, Fell topology, Hausdorff Metric topology, Wijsman topologies, Lower proximal locally finite topology,Locallyfnite topology, Poppe'sΔ-topologies, Uniformly discrete Hypertopology, Bounded topologies. 10 L

Hyperspace Topologies on Function Spaces [2]: Uniform convergence on compacta, equicontinuity, K-spaces, Proximal set-open topologies, Leader convergence and simple Leader convergence in function spaces, Quasi uniform convergence, Nearness convergence, Wijsman convergence, Proximal graph topologies on function spaces. 20 L

References

[1] Alejandro Wanes and Sam B. Nadler, Jr. Hyperspaces: Fundamentals and Recent Advances, Marcel Dekker, Inc. New York (1999).

[2] SomashekharNaimpally, Proximity Approach to Problems in Topology and Analysis, OldenbourgVerlag, Munich,Germany, (2009).

Suggested Readings

[1] James Dugundji, Topology, Allyn and Bacon, Inc., Boston (1966)

[2] John L. Kelly, General Topology, Springer-Verlag, New York Berlin Heideberg (1957).

[3] R. Lowen, Approach Spaces: The missing link in the Topology-Uniformity-Metric Triad, Clarendon Press, Oxford (1997).

MATH16-42 : Introduction to Greedy Appromixations

Greedy approximation, definition and examples. Quasi-greedy and almost greedy approximation. Lebesgue-type inequalities for greedy approximation. Saturation property of greedy-type algorithms. Approximation in compressed sensing: Equivalence of three approximation properties of the compressed sensing matrix, Construction of a good matrix, Exact recovery of sparse signals.

References

1. M. Fabian, P. Habala, P. Hajek, V.M. Santalucia, J. Pelant and V. Zizler, Functional Analysis and Infinite-Dimensional Geometry, Springer-Verlag, New York, 2001.

2. V. Temlyakov, Greedy Approximation, Cambridge University Press, 2011.